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ON THE $\bar{\partial}$-COHOMOLOGY GROUPS OF STRONGLY q-CONCAVE MANIFOLDS

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0. Introduction

Let X be a paracompact complex manifold of dimension n and $\pi: E \to X$ be a holomorphic vector bundle. We denote by $\Omega^p(E)$ the germ of E-valued holomorphic p-forms, and by $H^q(X, \Omega^p(E))$ the sheaf cohomology group of X of degree q with coefficients in $\Omega^p(E)$. In 1955, Serre showed the following basic theorem with respect to complex analysis.

Theorem (Serre duality, cf: [15]). *If $H^{q+i}(X, \Omega^p(E))$ $(i=0, 1)$ are Hausdorff, then $H^q(X, \Omega^p(E))$ is a Fréchet space, and its dual space and $H_k^{n-q}(X, \Omega^{n-p}(E^*))$ are isomorphic. Here, E^* denotes the dual of E, and $H_k^{\cdot}(X, \Omega^{\cdot}(E))$ denotes the compactly supported sheaf cohomology group of X with coefficients in $\Omega^{\cdot}(E)$.*

If $H^q(X, \Omega^p(E))$ is finite dimensional, then it is Hausdorff (cf: [15]). But, in general $H^q(X, \Omega^p(E))$ is not Hausdorff (cf: [8], [15]).

The cohomology groups of open manifolds were studied by Grauert [5] for solving Levi's problem, and his result played a fundamental role in the theory of singularities and hyperfunctions. As a natural extension of Grauert's work, it has been known that the finiteness of the cohomology groups results from on the convexity of manifolds:

X is called strongly q-convex (resp. strongly q-concave) if there exists an exhaustion function $\Phi: X \to \boldsymbol{R}$ of class C^∞ whose Levi form has at least $n-q+1$ positive (resp. $n-q+1$ negative) eigenvalues outside a compact subset K of X. We call K an exceptional set. In 1962, Andreotti and Grauert established finiteness theorems for cohomology groups which include the following theorem as a special case.