Torisu, I
Osaka J. Math.
33 (1996), 47-55

BOUNDARY SLOPES FOR KNOTS

ICHIRO TORISU

(Received November 7, 1994)

Let T be a torus. By the *slope* of an essential simple closed curve on T we mean its isotopy class. The *distance* $\Delta(r_1, r_2)$ between two slopes r_1 and r_2 is defined to be $|\gamma_1 \cdot \gamma_2|$, where γ_1 and γ_2 are curves with slopes r_1 and r_2 and $\cdot$ denotes homological intersection number. (Note that this is independent of all orientations. Note also that Δ is not a metric on the set of slopes ; the triangle inequality does not hold.)

Now let M be an irreducible, orientable 3-manifold and T a torus component of ∂M. Let $(F, \partial F) \subset (M, T)$ be an incompressible, boundary incompressible, orientable, genus g surface. Then the components of ∂F all have the same slope on T, and we call this the *boundary slope* of F. Let $S(M)_g$ denote the set of boundary slopes of such genus g surfaces. When M is an exterior $E(K)$ of a knot K, we write $S(E(K))_g$ as $S(K)_g$.

Gordon and Luecke gave estimations of ∂-slopes in $S(M)_0$ and $S(M)_1$, and showed that their estimations are the best possible (see [1], [3], [4]). So far, however, there is no estimation of ∂-slopes in $S(M)_g$ for $g \geq 2$.

In this paper, we give some estimation of ∂-slopes in $S(M)_g$ for arbitrary g when M has a certain geometric restriction, and we give an example which estimates the strength of the theorem.

Our main results are then the following.

Theorem 1. *If M has no essential annulus, then for any $g_1, g_2 \geq 1$, $r_1 \in S(M)_{g_1}$, $r_2 \in S(M)_{g_2}$, we have $\Delta(r_1, r_2) < 36(2g_1-1)(2g_2-1)$.*

Theorem 2. *Suppose a knot K has an m-string ∂-irreducible tangle decomposition.*

(i) *Let a/b ($\neq 0/1$) be an element of $S(K)_g$, where a and b are coprime integers. Then $|b| \leq g/m$.*

(ii) *$g(K) \geq (m+1)/2$, where $g(K)$ is the genus of K.*

Theorem 3. *For any n non-trivial knots $K_1, \cdots, K_n$ and $a/b \in S(K_1 \# \cdots \#$*