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MODULI OF EQUIVARIANT ALGEBRAIC VECTOR BUNDLES OVER AFFINE CONES WITH ONE DIMENSIONAL QUOTIENT

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Introduction

Let G be a reductive complex algebraic group. We consider on the base field $\mathbf{C}$ of complex numbers. Let X be an affine G-variety with a G-fixed base point $x_0 \in X$ and Q be a G-module. We denote by $Vec_G(X,Q)$ the set of algebraic G-vector bundles over X whose fiber at x_0 is Q and by $VEC_G(X,Q)$ the set of G-isomorphism classes in $Vec_G(X,Q)$. The set $VEC_G(X,Q)$ has the distinguished element represented by the product bundle $\Theta_Q := X \times Q$. We denote by $[E]$ the isomorphism class of $E \in Vec_G(X,Q)$.

The study of $VEC_G(X,Q)$ is especially interesting when X is a G-module P (see e.g. [2]). In this case we take the origin as the G-fixed base point. When G is trivial, the Serre conjecture, which was proved by Quillen and Suslin independently, implies that $VEC_G(P,Q)=\{*\}$ (the trivial set consisting of the distinguished element) for any P and Q. However, only few facts are known when G is non-trivial. One approach is to require that the quotient space $P//G$ be of small dimension. It is easy to see that $VEC_G(P,Q)=\{*\}$ if $\dim P//G=0$. But, $VEC_G(P,Q)$ is not trival in genėral. Schwarz [11] (see [5] for the details) has shown that if $\dim P//Q=1$, $VEC_G(P,Q)$ has a structure of finite dimensional vector group and it can be non-trivial. Later, many other families of non-trivial examples have been produced by Knop [4], Masuda-Petrie [9] and Masuda-Moser-Petrie [7] when P has a higher dimensional quotient. However it remains open to classify elements in $VEC_G(P,Q)$ when $\dim P//G \geq 2$.

If $\dim P//G \geq 1$, there is a non-zero point $x \in P$ whose orbit is closed. The closure of the orbit of the line spanned by x is an affine cone with G-action whose quotient is one dimensional (but not necessarily isomorphic to affine line). Masuda-Moser-Petrie [8] noticed that elements of $VEC_G(P,Q)$ can be often distinguished by restricting to the cone. This led them to the notion of *weighted G-cones with smooth one dimensional quotient* (see §1). Note that a G-module with one dimensional quotient is an example of a weighted G-cone with smooth one dimensional quotient.