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KURANISHI FAMILY OF STRONGLY PSEUDO-CONVEX DOMAINS

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Introduction

The purpose of this paper is to complete T. Akahori's construction of the semi-universal family of strongly pseudo-convex (s.p.c. for abbreviation) domains of $\dim_C X \geq 4$ (cf. [1], [2]), in the general deformation theoretic context. In [6], J. Bingener and S. Kosarew considered deformations of s.p.c. complex spaces, and they proved the existence of the semi-universal family for deformations as germs along a compact subspace with a certain property (in the case of a non-singular complex space, along the exceptional subset) and conjectured the exstence of a formally semi-universal convergent family for deformations as whole complex spaces. In this paper, we will consider deformations of s.p.c. manifolds and will show that the Akahori's canonical family of complex structures on a compact level subset of a strictly pluri-subharmonic exhaustion function (cf. [1], [2]) induces both the semi-universal family and the formally semi-universal converent family as above, if $\dim_C X \geq 4$.

The key step is to construct the semi-universal family for deformations as germs along a compact level subset from the Akahori's canonical family of complex structures. Though the correspondence between families of complex structures and of complex manifolds is not direct, it is rather simple if we restrict ourselves in the classical deformation theory. Let r be a strictly pluri-subharmonic exhaustion function on a s.p.c. complex manifold X with $\dim_C X \geq 4$. We denote $\Omega_\varepsilon = \{x \in X \mid r(x) < \varepsilon\}$ and suppose $K = \cap_{\alpha<\varepsilon} \Omega_\varepsilon$ for some $\inf_X r \leq \alpha < \sup_X r$. Consider a fibred groupoid $p: F_K \to \mathscr{C}$ (the category of germs of complex spaces) of deformations of X as germs along K and its restriction over $\mathscr{C}_{\text{red}}$ (the category of germs of reduced complex spaces) $p_{\text{red}}: (F_K)_{\text{red}} \to \mathscr{C}_{\text{red}}$. In [1] and [2], Akahori constructed a family of complex structures over each $\bar{\Omega}_\varepsilon$ ($\alpha < \varepsilon$) which is effective and complete in the sense that any family of deformations of a neighbourhood of $\bar{\Omega}_\varepsilon$ is induced from that family of complex structures over $\bar{\Omega}_\varepsilon$ (what is called "*versal in the sence of Kuranishi*" in [1]). This implies that, for each $\varepsilon > \alpha$, there exists a family $\mathscr{X}_\varepsilon \to T_\varepsilon$

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