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CONSTRUCTION OF c_2-SELF-DUAL BUNDLES ON A QUATERNIONIC PROJECTIVE SPACE

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1. Introduction

In the A.D.H.M.-construction of anti-self-dual bundles on S^4, the Penrose twistor transformation $\pi_Z: \boldsymbol{P}^3 \to S^4$ plays an important role [2],[6]. This fibration is easily obtaind when we identify S^4 with $\boldsymbol{HP}^1$ and $\boldsymbol{C}^2$ with $\boldsymbol{H}$. An anti-self-dual vector bundle F with unitary structure on S^4 is lifted to a holomorphic vector bundle $\tilde{F}$ on $\boldsymbol{P}^3$ with respect to the induced connection. $\boldsymbol{P}^3$ has a real structure σ induced by the right multiplication by the unit quaternion j which is different from the usual real strucure and preserves the fibration. Although σ on $\boldsymbol{P}^3$ has no fixed points, it does have fixed lines. These are precisely the fibres of π_Z. So the fibres of π_Z are called *real lines* and are denoted by $\boldsymbol{P}_x$ ($x \in S^4$). Then the above holomorphic vector bundle $\tilde{F}$ has the following properties.

(1) $\tilde{F}$ restricted to an arbitrary real line $\boldsymbol{P}_x$ is holomorphically trivial.

(2) The cohomology group $H^1(\boldsymbol{P}^3, \tilde{F}(-2))$ vanishes.

By the definition of $\tilde{F}$, (1) is trivial. But (2) is a deep result given by Drinfeld-Manin [6], Rawnsley [16], Douady [5] and Hitchin [8]. On the other hand, Barth-Hulek [3] have shown the following. If a holomorphic vector bundle E on $\boldsymbol{P}^3$ satisfies $E \cong E^*$(E^* is the dual bundle) and $H^1(\boldsymbol{P}^3, E(-2))=0$ and if E restricted to some line is holomorphically trivial, then E is constructed by some monad. Consequently, A.D.H.M-construction is completed.

We take $\boldsymbol{HP}^n$ instead of $S^4 \cong \boldsymbol{HP}^1$. Differential-geometrically, $\boldsymbol{HP}^n$ is one of the quaternionic Kähler manifolds which are $4n$-dimensional oriented Riemannian manifolds whose holonomy groups are contained in the subgroup $Sp(n)\cdot Sp(1), n \geqq 1$. Nitta [13] and Mamone Capria and Salamon [4] have developed independently higher dimensional analogues of the notion of (anti-)self-dual connections on a quaternionic Kähler manifold. Those connections are called c_1,c_2 and c_3-self-dual connections in Galicki and Poon [7] which are Yang-Mills. We use these terminology.

As for the Penrose twistor space on a half-conformally flat manifold [1], we also have a higher dimensional analogue. Salamon showed that there is a twistor space Z which has a natural complex structure on an arbitrary quaternionic Kähler manifold M [18]. If we pull back c_2-self-dual form to Z, we get (1,1) form on