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THE FIXED POINT SET OF *C* ACTIONS ON A COMPACT COMPLEX SPACE

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Introduction

Let X be a connected complete algebraic variety defined over the complex number field C. Suppose that a connected unipotent linear algebraic group G acts regulary on X. Let X^G be the set of fixed points of this action. Then in [3] Horrocks has shown that X^G is connected and that the inclusion $X^G \hookrightarrow X$ induces an isomorphism of the algebraic fundamental groups, i.e., the profinite completions of the fundamental groups, of both spaces. In the complex analytic category, by Carrell and Sommese this result was partially generalized in [1], where they have shown that if X is a general connected complex manifold and G is a unipotent algebraic group as above which acts biholomorphically and meromorphically on X (cf. 1.2 for the definition), then the set of fixed points X^G is connected.

In this note we push this analogy a little further to one on the compartison of the fundamental groups. Namely, we show that when X is a compact complex space, the inclusion $X^G \hookrightarrow X$ induces and isomorphism of the (topological) fundamental groups of these spaces (cf. Theorem 3.1).

1. Preliminaries

In this section we summarize some of the known results on the fundamental groups and the meromorphic actions of algebraic groups on a complex space.

1.1. First we consider the fundamental groups. Let X be a topological space. We denote by $\pi_0(X)$ (resp. $\pi_1(X)$) the set of connected components (resp. the fundamental group with respect to some reference point) of X. (We shall be sloppy about the base points of fundamental groups throughout the paper.) We shall use the following terminology:

DEFINITION 1.1. Let $f: X \to Y$ be a continuous map of X into another topological space Y. Then we say that

1) f is *0-connected* if the induced map $f_{0*}: \pi_0(X) \to \pi_0(Y)$ is bijective, and