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THE SUBMANIFOLD OF SELF-DUAL CODES IN A GRASSMANN MANIFOLD

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1. Introduction

By a $[N,m]$-linear code over a finite field F, we mean an m-dimensional vector subspace of an N-dimensional vector space V over F. Let $C^{\perp}$ be the orthogonal complement of a $[N,m]$-linear code C in V, that is $C^{\perp}=\{v\in V|\langle v,c\rangle=0 \text{ for any } c\in C\}$, where $\langle\ ,\ \rangle$ denotes a fixed inner product of V. This is called the dual code of C which is a $[N,N-m]$-linear code. C is called self-orthogonal (resp. self-dual) if and only if $C\subset C^{\perp}$ (resp. $C=C^{\perp}$). For any linear code, it may be known that there exists a self-dual embedding, and so every linear code can be made from a self-dual code. Therefore we are interested in self-dual codes. Since a linear code C is a vector space, C can be thought as an element of the Grassmann manifold $GM(m,V)$. Similarly, $C^{\perp}$ can be thought as an element of $GM(N-m,V)$. As a set, $GM(m,V)$ and $GM(N-m,V)$ are isomorphic so that C and $C^{\perp}$ correspond each other as elements of the Grassmann manifolds. In this paper, we shall study the self-orthogonality and the self-duality of linear codes through the Grassmann manifolds. In section 1, we shall give a constructive proof of self-dual embedding of linear codes. In section 2, we shall summarize about the Grassmann manifolds and give an elementary result about the self-duality using a projective embedding. In section 3, we shall give our main theorem on self-orthogonality and self-duality of linear codes. This theoerm shows that self-orthogonal codes and self-dual codes are on a quadratic surface in the projective space. Combining our results, we can see that every linear code can be obtained from a self-dual code, and every self-dual code is a special case of a self-orthogonal code.

2. Self-dual embedding of linear codes

In this section, we assume $N=n+m$. Let C be a $[N,m]$-linear code over a finite field F. We shall construct a self-dual code which contains C as an embedding image. It may be known, but this is a motive for studying self-dual codes and so we shall give the proof. Since C can be thought as a subspace of