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UNIFORM ESTIMATES FOR FUNDAMENTAL SOLUTIONS ASSOCIATED WITH NON-LOCAL DIRICHLET FORMS

Dedicated to Professor Masatoshi Fukushima for his 60th birthday

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0. Introduction

The generator of a Markov process of jump type is a pseudo-differential operator of non-local type. If the pseudo-differential operator satisfies a certain regularity condition, the fundamental solution of the parabolic equation associated with the operator can be constructed by the general theory of pseudo-differential operators, and properties of the solution are investigated by the theory ([8] Chap.7 §4, [9], [11]). However, in the case where the elements of the operator are discontinuous, we need another theory.

Any symmetric stable process is associated with a Dirichlet form of non-local type. In Komatsu [7] we proved that, if a Dirichlet form is bounded from above and below by Dirichlet forms associated with stable processes of the same index, there exists a strong Feller process associated with the Dirichlet form. We shall call the strong Feller process a stable type process. More specifically, we obtained some uniform estimates for the transition functions of stable type processes which are fundamental solutions of parabolic equations in the weak sense associated with Dirichlet forms of non-local type. We note that Carlen-Kusuoka-Stroock [3] studied upper bounds for transition functions in a more ganeral context.

In this paper, we still work with the transition functions of stable type processes and present, among others, a lower estimate and a uniform Hölder estimate for them, which we were unable to obtain in the previous paper [7]. Our lower estimate is almost the same as the one naturally expected from the Aronson estimate in [1] and [2]. These results can be proved through some improvements of the proof in [7]. Actually we employ a wider class of stable type processes than [7] and prove that the uniform estimates of their transition functions similar to those in [7] remain valid for this enlarged class. Finally we examine those examples where the Dirichlet forms are expressed as integrals of bilinear forms involving pseudo-differential operators. This type of forms has been considered by Jacob ([4], [5], [6]). An example of this type will indicate the necessity of