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SELF-DUAL GENERALIZED TAUB-NUT METRICS

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0. Introduction

The Taub-NUT metrics were found by Taub [10] in 1951, and extended by Newmann-Unti-Tamburino [8] in 1963. They are Ricci-flat self-dual metrics on $\boldsymbol{R}^4$ and were given by Hawking [4] as a non-trivial example of the gravitational instantons which he advocated in 1977. Iwai-Katayama [5] generalized the Taub-NUT metrics in the following way. Suppose that a metric $\breve{g}$ on an open interval U in $(0, +\infty)$ and a family of Berger metrics $\hat{g}(r)$ on S^3 indexed by U are given, where a Berger metric is by definition a right invariant metric on $S^3 = Sp(1)$ which is further left $U(1)$-invariant. Then the twisted product $g = \breve{g} + \hat{g}(r)$ on the annulus $U \times S^3 \subset \boldsymbol{R}^4 \backslash \{0\}$ is called a generalized Taub-NUT metric. In [5] they explicitly wrote out the conformally flat ones among those metrics and gave a condition for a generalized Taub-NUT metric to be self-dual in terms of a certain ordinary differential equation.

In this paper we will solve this differential equation and explicitly express all the self-dual generalized Taub-NUT metrics. As an application we get a family of complete Einstein self-dual generalized Taub-NUT metrics on 4-balls. This reproduces the Einstein self-dual metrics found by Pedersen [9] which have Berger spheres as conformal infinities.

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1. Calculations of the curvatures

Let $\boldsymbol{R}^4$ denote the Euclidean 4-space with the standard metric endowed with the standard orientation. Let (x, y, z, w) be the standard coordinates on $\boldsymbol{R}^4$ and r the square of the distance from the origin, that is, $r = x^2 + y^2 + z^2 + w^2$. Let us introduce a positively oriented orthonormal frame of the cotangent bundle of $\boldsymbol{R}^4 \backslash \{0\}$ as follows: