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ON A THEOREM OF ZARISKI - VAN KAMPEN TYPE AND ITS APPLICATIONS

TAKANORI MATSUNO

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1. Introduction. Zariski constructed a method to calculate $\pi_1(\boldsymbol{P}^2-C)$, where $\boldsymbol{P}^2$ is the complex projective plane and C is a curve on it. In this paper, following the ideas of Zariski [5] and Van Kampen [4], we give a method to calculate $\pi_1(E-S)$, where E is a holomorphic line bundle over a complex manifold M and S is a hypersurface of E under certain conditions. Applying our method and the Reidemeister-Schreier method (see Rolfsen [3]), we can calculate the fundamental groups of regular loci of certain normal complex spaces. We give a few concrete examples in the final section.

This paper is a revised version of the author's master thesis [1]. The author would like to express his thanks to Professor M. Namba for his useful suggestions and encouragements and to Professor M. Sakuma whose suggestions about Lemma 1 (see section 2) was a great help to prove Main Theorem. He also expresses his thanks to the referee for useful comments.

2. Statement of Main Theorem. Let M be a connected -dimensional complex manifold and $\mu: E\to M$ be a holomorphic line bundle over M and S be a hypersurface of E. We assume that E and S satisfy the following conditions:

(1) $\mu: S\to M$ is a finite proper holomorphic map, where μ' is the ristriction of μ to $S(\mu'=\mu|s)$.

(2) There is a hypersurfase B of M such that $\mu'|_{S-\mu^{-1}(B)}: S-\mu^{-1}(B)\to M-B$ is an unbranched covering of degree .

(3) $(\mathrm{d}\mu')_p: T(S-\mu^{-1}(B))_p\to T(M-B)_{\mu'(p)}$ is isomorphic for every point $p\in S-\mu^{-1}(B)$.

Then we have a following lemma whose proof is given in section 4.

Lemma 1. $\mu|_{E-S-\mu^{-1}(B)}: E-S-\mu^{-1}(B)$ *is a continuous fiber bundle.*

We denote a standard fiber od $\mu: E\to M$ by $\hat{F}$ and that of $\mu|_{E-S-\mu^{-1}(B)}: E-S-\mu^{-1}(B)\to M-B$ by F. We assume that there is a continuous section $\xi: M\to E$ of $\mu: E\to M$ such that $\xi(M)\cap S=\phi$ (see Figure 1).