Fujisawa, T
Osaka J. Math.
32 (1995), 613-636

ON NON-RATIONAL NUMERICAL DEL PEZZO SURFACES

TARO FUJISAWA

(Received Octover 22, 1993)

Introduciton

In this paper we call a normal compact complex surface X a numerical Del Pezzo surface if X is a Moishezon surface, the intersection number $(-K_X)\cdot C$ is positive for every curve C on X, and the self-intersection number $(-K_X)^2$ is positive (see Definition 1.1). If X is nonsingular, such a surface is called a Del Pezzo surface and its properties are fairly well-known. Several results on such surfaces are obtained by F. Hidaka and K.-i.Watanabe [9] when X is Gorenstein, by F. Sakai [13] when X is rational $\boldsymbol{Q}$-Gorenstein, and by L. Badescu [3] when X is non-rational $\boldsymbol{Q}$-Gorenstein.

The purpose of this paper is to study the structure of non-rational numerical Del Pezzo surfaces. In the present paper the canonical divisor K_X is not necessarily $\boldsymbol{Q}$-Cariter. In section 1 we define the notion of a numerical Del Pezzo surface and study its basic properties. In section 2 we describe the structure of a non-rational numerical Del Pezzo surface (Theorem 2.1). Our results are similar to those in L. Badescu [3], where the surface is assumed to be $\boldsymbol{Q}$-Gorenstein. In section 3 we define the notion of a minimal contraction of a ruled surface, and the notion of a DP1-ruled surface, which is a ruled surface whose singular fibers are of special type (see Definition 3.2 and 3.7). Then we obtain a criterion for the minimal contraction of a DP1-ruled surface to be a non-rational numerical Del Pezzo surface (Theorem 3.11). This is one of the main results of this paper. In section 4 we define the notion of indices of non-rational numerical Del Pezzo surfaces, and show that they are the minimal contractions of DP1-ruled surfaces if the Picard numbers are equal to 1 and their indices are prime numbers (Theorem 4.9). In appendix we prepare several results on weighted graphs. As for terminologies on weighted graphs, the reader may consult T. Fujita [7] and P. Orlik-P. Wagreich [12].

The author would like to express his gratitude to Professor S. Ishii and Professor T. Fujita for their helpful advice and encouragement.