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ON RINGS WHOSE CYCLIC FAITHFUL MODULES ARE GENERATORS

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0. Introduction

For each positive integer n, we temporarily say that a ring R is n-PF (n-pseudo-Frobenius) if every faithful right R-module generated by at most n elements is a generator for the category of all right R-modules. As well known, the ring which is n-PF for all positive integers n is called a right FPF (finitely pseudo-Frobenius) ring, and every FPF ring splits in a ring with essential singular ideal and a nonsingular ring. Nonsingular FPF rings were investigated in S. Kobayashi [9] and S. Page [11], [12], [13], etc.; in particular, S. Page [11] characterized (von Neumann) regular right FPF rings as self-injective regular rings having bounded index, and S. Kobayashi [9] gave a characterization of nonsingular right FPF rings. The aim of this paper is to study nonsingular 1-PF rings, which were to some extent investigated in G.F. Birkenmeier [2], [3] and S. Kobayashi [10].

Modifying the proof of [10, Proposition 1] and observing that the converse of the proposition is also true, we see, as will be noted in §3, that for a fixed integer $n \geq 2$, a ring R is right nonsingular and n-PF if and only if R satisfies the condition (C_n) that:

(i) R is right bounded, i.e., every essential right ideal of R contains a two-sided ideal which is essential in R as a right ideal,

(ii) For every right ideal A generated by at most n elements, $R = Tr_R(A) \oplus r_R(A)$, where $Tr_R(A)$ (respectively, $r_R(A)$) is the trace (resp. the right annihilator) ideal of A, and

(iii) Every nonsingular right R-module generated by at most n elements can be embedded in a free right R-module.

However, such the result as above is, in general, false in the case $n=1$. Moreover, for regular or commutative semiprime rings, the FPF condition is, as noted in [10], equivalent to the n-PF condition for each $n \geq 2$, although it is not