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ARTINIAN RINGS RELATED TO RELATIVE ALMOST PROJECTIVITY II

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Let R be an artinian ring. In [10], we have studied R on which the following condition holds: for R-modules M and N, if M is N-projective, then M' is always almost N-projective for every submodule M' of M. If M' is always N-projective in the above, then this property characterizes hereditary rings with $J^2=0$ [2] and [6], where J is the Jacobson radical of R.

We have investigated the above condition in [10], when i); M and N are local and ii): M is local and N is a direct sum of local modules. In this paper we give a characterization of R over which the above condition is satisfied for any R-modules M and N.

1. Preliminaries

In this paper R is always an artinian ring with identity, and every module is a finitely generated R-module. We shall use the same notations given in [10].

We have studied rings R over which the following condition is satisfied in [10]:

For any R-modules M and N

(#) if M is N-projective, then M' is always almost N-projective for every submodule M' of M.

We denote primitive idempotents in R by e, f, g, and so on. Assume that (#) holds whenever M and N are local. Then we have shown in [10] that R has the following structure:

$J^3=0$ and for a primitive idempotent e with $eJ^2\neq 0$

(0) $eR\supset eJ\simeq\sum_K\oplus(f_kR)^{(n_k)}\oplus\sum_J\oplus S_j$,

where the f_iR is a uniserial and projective module with $f_iJ^2=0$, $f_iJ\neq 0$ and the S_j is simple.

(If necessary, we use the following decomposition:

$e_iR\supset e_iJ\approx\sum_{K(1)}\oplus(f_{ik}R)^{(n_{ik})}\oplus\sum_{J(1)}\oplus S_{ij}$.)

We shall use frequently the following theorem: [10], Theorem 1.

Theorem 0. *Let R be artinian. Then (#) holds whenever M and N are local*