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INTEGRAL REPRESENTATIONS OF UNRAMIFIED GALOIS GROUPS AND MATRIX DIVISORS OVER NUMBER FIELDS

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Introduction

The purpose of the present note is to pursue some analogies for number fields after the model of A. Weil's work [13].

Weil ([13]) studied the space of representations of the fundamental group of a curve and showed it has a structure of an algebraic variety, as a generalization of the Jacobian variety, by employing his generalized Riemann-Roch theorem for matrix divisors. He used the Poincaré zeta (theta) Fuchs functions in order to attach matrix divisor classes to representations of the fundamental group. This process may be regarded as a sort of non-abelian Kummer theory. Actually, we may observe that the "Poincaré sum" in Hilbert's theorem 90 plays an analogous role to the Poincaré zeta (theta) Fuchs function.

Following the principle on the analogy between number fields and function fields ([14]), we would like to discuss some analogies, for number fields, of the function field case described as above. In Section 1, we introduce matrix divisors for number fields. A version of the Riemann-Roch theorem for them is then known as the Poisson summation formula ([11], 4.2). So, this section has totally expository nature. In Section 2, we will attach matrix divisor classes to integral, unitary representations of unramified Galois groups by means of Hilbert's theorem 90 for the general linear group and see some general properties. In Section 3, we discuss an example.

NOTATION. $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ denote the ring of rational integers, the fields of rational, real and complex numbers respectively. For a number field K of finite degree over $\boldsymbol{Q}$, we use the following notation.
$\mathcal{O}_K$: =the ring of integers in K.
X_K : $=\mathrm{Spec}(\mathcal{O}_K)$ together with the structure sheaf $\mathcal{O}_{X_K}$
X_K°: =the set of closed points of X_K=the set of finite places of K.
X_K^∞ : =the set of complex conjugation classes of the embeddings of K into $\boldsymbol{C}$=the set of infinite places of K.