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ON STANDARD L-FUNCTIONS ATTACHED TO $\mathrm{ALT}^{n-1}(C^n)$-VALUED SIEGEL MODULAR FORMS

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Introduction

In [23], we studied some properties of standard L-functions attached to $\mathrm{sym}^l(V)$-valued Siegel modular forms of weight $\det^k\otimes\mathrm{sym}^l$. More precisely, let $\det^k\otimes\mathrm{sym}^l$ be an irreducible rational representation of $GL(n, \boldsymbol{C})$ with representation space $\mathrm{sym}^l(V)$, where V is isomorphic to $\boldsymbol{C}^n$ and $\mathrm{sym}^l(V)$ is the l-th symmetric tensor product of V. Let f be a $\mathrm{sym}^l(V)$-valued holomorphic cusp form of weight $\det^k\otimes\mathrm{sym}^l$ for $Sp(n, \boldsymbol{Z})$ (size $2n$). Suppose f is an eigenform, i.e., a non-zero common eigenfunction of the Hecke algebra. Then we define the standard L-function attached to f by

$$(0.1)\qquad L(s, f, \underline{\mathrm{St}}):=\prod_p\left\{(1-p^{-s})\prod_{j=1}^{n}(1-\alpha_j(p)^{-1}p^{-s})(1-\alpha_j(p)p^{-s})\right\}^{-1},$$

where p runs over all prime numbers and $\alpha_j(p)$ $(1\leq j\leq n)$ are the Satake p-parameters of f. The right-hand side of (0.1) converges absolutely and locally uniformly for $\mathrm{Re}(s)>n+1$. We put

$$\Lambda(s, f, \underline{\mathrm{St}}):=\Gamma_{\boldsymbol{R}}(s+\varepsilon)\Gamma_{\boldsymbol{C}}(s+k+l-1)\left\{\prod_{j=2}^{n}\Gamma_{\boldsymbol{C}}(s+k-j)\right\}L(s, f, \underline{\mathrm{St}}),$$

with

$$\Gamma_{\boldsymbol{R}}(s):=\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right),\ \Gamma_{\boldsymbol{C}}(s):=2(2\pi)^{-s}\Gamma(s)$$

and

$$\varepsilon:=\begin{cases}0 \text{ for } n \text{ even},\\ 1 \text{ for } n \text{ odd}.\end{cases}$$

Then we have the following (cf. Andrianov and Kalinin [Z], Böcherer [5] and Mizumoto [19] for $l=0$).