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ON REGULAR RINGS WHOSE MAXIMAL RIGHT QUOTIENT RINGS ARE TYPE I_f

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Introduction

This paper is written about the property (DF) on regular rings whose maximal right quotient rings are Type I_f. Hereafter regular rings whose maximal right quotient rings are Type I_f are said to satisfy (*). The property (DF) is very important property when we study on regular rings satisfying (*), and it was treated in the paper [5] written by the first author, where (DF) for a ring R is defined as that if the direct sum of any two directly finite projective R-modules is always directly finite. In the above paper, the equivalent condition that a regular ring R of bounded index satisfies (DF) was discovered and called (♯). Stillmore, we proved that the condition (DF) is equivalent to (♯) for regular rings whose primitive factor rings are artinian in the paper [6]. Then we have the problem that (DF) is equivalent (♯) for regular rings satisfying (*) or not, where the condition (*) is weaker than one that primitive factor rings are artinian.

In § 2, we shall prove Theorem 2.4. This is important, and using this, Theorem 2.5 (i.e. if R is a regular ring satisfying (*) and k is any positive integer, then kP is directly finite for every directly finite projective R-module P) is proved. Moreover, we shall solve the above problem in Theorem 2.11.

In § 3, we shall consider some applications of Theorem 2.11. We prove Theorem 3.3 that if R is a regular ring satisfying (*) whose maximal right quotient ring of R satisfies (DF), then so does R. Though it is clear that a regular rings satisfying (*) which has a nonzero essential socle satisfies (DF), we can prove that, for regular rings satisfying (*), the condition having a nonzero essential socle is not equivalent to (♯) in Example 3.4. Next, we shall consider that $(\Pi_1^\infty R)/(\oplus R)$ satisfies (DF) or not for a regular ring R satisfying (*). This problem is a generalization of Example 3.4, and we prove that, for a regular ring R of bounded index, $(\Pi_1^\infty R)/(\oplus R)$ satisfies (DF) (Theorem 3.9).

Throughout this paper, R is a ring with identity and R-modules are unitary right R-modules.