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REMARKS ON AN EXCLUSIVE EXTENSION GENERATED BY A SUPER-PRIMITIVE ELEMENT

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Throughout this paper, all fields, rings and algebras are assumed to be commutative with unity. Our special notations are indicated below, and our general reference for unexplained technical terms is [3].

Let R be a Noetherian domain and K its quotient field. Let α be an algebraic element over K with the minimal polynomial $\varphi_\alpha(X)=X^d+\eta_1X^{d-1}+\cdots+\eta_d$. In [6], we have shown that if $R[\alpha]\cap K=R$ then $\bigcap_{i=1}^{d-1}I_{\eta_i}\subseteq I_{\eta_d}$. Our objective of this paper is to show the converse of this result under certain assumptions, which will be established in Theorem 5.

In what follows, we use the following notations unless otherwise sepecified:

R: a Noetherian integral domain,
$K:=K(R)$: the quotient field of R,
L: an algebraic field extension of K,
α: a non-zero element of L,
$d=[K(\alpha):K]$,
$\varphi_\alpha(X)=X^d+\eta_1X^{d-1}+\cdots+\eta_d$, the minimal polynomial of α over K.
$I_{[\alpha]}:=\bigcap_{i=1}^{d}(R:_R\eta_i)$, which is an ideal of R.
$I_a:=R:_RaR$ for $a\in K$.

It is clear that for $a\in K$, $I_{[a]}=I_a$ from difinitions.

$J_{[\alpha]}:=I_{[\alpha]}(1,\eta_1,\cdots,\eta_d)$,
$\tilde{J}_{[\alpha]}:=I_{[\alpha]}(1,\eta_1,\cdots,\eta_{d-1})$.

We also use the following standard notation :

$Dp_1(R):=\{p\in Spec(R)|depth R_p=1\}$.

Let R be a Noetherian domain and K its quotient field. Take an element α in an field extension of K. When $R[\alpha]\cap K=R$, we say that α is an *exclusive* element over R and that $R[\alpha]$ is an *exclusive extension* of R. Let $\pi:R[X]\to R[\alpha]$ be the R-algebra homomorphism sending X to α. The element α is called an *anti-integral* element of degree d over R if $Ker\,\pi=I_{[\alpha]}\varphi_\alpha(X)R[X]$. When α is an anti-integral element over R, $R[\alpha]$ is called an *anti-integral extension* of R. For $f(X)\in R[X]$,