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NOTES ON TRIVIAL SOURCE MODULES

Dedicated to Professor S. Endo on his 60th birthday

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1. Introduction

Let G be a finite group and k an algebraically closed field of characteristic p. An indecomposable kG-module with a vertex Q is said to be *a weight module* if its Green correspondent with respect to $(G,Q,N_G(Q))$ is simple. Let B be a block of kG. Alperin [1] conjectured that the number of the weight modules belonging to B equals that of the simple modules in B. If this is the case and a defect group of B a TI set, then it can be shown under some additional assumption that the socles of weight modules are simple, which in turn determine the isomorphism classes of the weight modules ; this holds if G is a simple group with a cyclic Sylow p-subgroup. This rather surprising property has been known to hold for finite groups of Lie type of characteristic p. However little is known about general properties of weight modules. In the final section we shall study solvable groups that have only simple weight modules.

Throughout this paper G denotes a finite group and k an algebraically closed field of prime characteristic p. For a kG-module M, hd(M), soc(M) and P(M) denote the head, socle and projective cover of M respectively. If N is a kG-module, $N|M$ indicates that N is isomorphic to a direct summand of M, and (N,M) denotes the multiplicity of N as a summand of M. We fix a block B of kG and let D be its defect group. IRR(B) denotes a full set of non-isomorphic simple modules in B,$l(B)$ its cardinality and WM(B) a full set of non-isomorphic weight modules belonging to B.Let f be the Green correspondence with respect (G,D,H), where $H=N_G(D)$. If WM($B|D$) denotes the subset of WM(B) consisting of the weight modules with vertices D and b the Brauer correspondent of B in kH, then f induces a bijection between WM($B|D$) and IRR(b).

The author thanks the referee for improving the proof of Proposition 4 below.

2. Weight modules over blocks with TI defect groups

To begin with, we quote the following as a preliminary lemma.

Lemma 1 (Robinson [8]). *Let T be a subgroup of G. Let M (resp. N) be*