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ON AN ALGORITHM FOR CONSTRUCTING MULTI-DIMENSIONAL WAVELETS

Dedicated to Professor Tosinobu Muramatu on the occasion of his sixtieth birthday

RYUICHI ASHINO, MICHIHIRO NAGASE and RÉMI VAILLANCOURT

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1. Introduction

Let $\{V_j\}_{j\in \boldsymbol{Z}}$ be an r-regular multiresolution analysis in $L^2(\boldsymbol{R}^n)$ and $\varphi(x)$ an r-regular father function. Denote by $\boldsymbol{T}$ the one-dimensional torus $\boldsymbol{R}/2\pi\boldsymbol{Z}$. Then there exists an isomorphism of Hilbert spaces between V_0 and $L^2(\boldsymbol{T}^n)$,

$$V_0 \ni f \leftrightarrow m_f \in L^2(\boldsymbol{T}^n), \tag{1}$$

defined by the functional equations

$$\hat{f}(2\xi) = m_f(\xi)\hat{\varphi}(\xi) \tag{2}$$

and

$$m_f(\xi) = 2^{-n} \sum_{k\in \boldsymbol{Z}^n} \left(f\left(\frac{x}{2}\right), \varphi(x-k) \right) e^{-ik\cdot\xi}. \tag{3}$$

Here $\hat{f}(\xi)$ denotes the Fourier transform of $f(x)$ and $(\cdot,\cdot)$ denotes the inner product of $L^2(\boldsymbol{R}^n)$. The function $m_f(\xi)$ will be called the *symbol* of $f(x)$.

Put $R=\{0,1\}^n$ and $E=R\backslash(0,\cdots,0)$. To construct (2^n-1) mother functions $\psi_\varepsilon(x)$, $\varepsilon\in E$, we need to construct $2\pi\boldsymbol{Z}^n$-periodic L^2-functions, m_{ψ_ε}, which satisfy conditions to be specified below. For simplicity, we write m_0 for m_φ and m_ε for $m_{\psi_\varepsilon}, \varepsilon\in E$.

To show that the mother functions $\psi_\varepsilon(x)$ are r-regular, it is sufficient to show that m_ε satisfy the same property as m_0. Therefore the simpler the construction of m_ε, the better it is.

As asserted by Meyer [2, Section 3.4, Corollary 2], the functions $\varphi(x-k)$ and $\psi_\varepsilon(x-k)$, $\varepsilon\in E, k\in \boldsymbol{Z}^n$ form an orthonornal basis of V_1 if and only if the $2^n\times 2^n$ matrix

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