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ON COMPATIBLE REGULARIZING DATA FOR SECOND ORDER HYPERBOLIC INITIAL-BOUNDARY VALUE PROBLEMS

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1. Introduciton.

It is well known that a necessary condition to solve an initial-boundary value problem in a proper domain of $\boldsymbol{R}^n$ is that the data of the problem satisfy, at the boundary of the domain, compatibility conditions of a certain order, which generally depends on the regularity assumed of the data, and required of the solution. In many situations, one is led to consider approximations of the solutions, obtained by solving problems with more regular data; thus, one needs to construct more regular data that not only approximate the given ones, but also satisfy compatibility conditions of higher order. A typecal example occurs when, in order to prove the existence of a solution to the original problem by means of energy methods, one first establishes the required energy estimates on more regular solutions (which it is possible to differentiate), and then resorts to a density argument. This is, for instance, the method followed by Ikawa, [8], and Shibata, [16], for linear hyperbolic equations of second order with Neumann type boundary conditions, and by Dan, [6], for a linear coupled hyperbolic-parabolic system, again with Neumann boundary conditions. A similar situation was considered by Rauch and Massey, [15], while proving the regularity of solutions to a linear first order hyperbolic system, under general boundary conditions.

More recently, Beirao DaVeiga ([1,2,3,4,5]) presents and develops a general method to prove the strong continuous dependence with respect to the data of solutions to nonlinear hyperbolic problems, including the nonlinear Neumann problems considered by Shibata-Kikuchi, [18], and Shibata-Nakamura, [19], as well as several systems of nonlinear fluid dynamics; in particular, the model nonlinear Neumann problem

$$\begin{aligned} & u_{tt}-\operatorname{div} A(\nabla u)=f(x,t) && \text{in } \Omega\times]0,T[\\ & u(x,0)=u_0(x),\ u_t(x,0)=u_1(x) && \text{in } \Omega \\ & \nu\cdot A(\nabla u)+b(u)=\phi(x,t) && \text{in } \partial\Omega\times]0,T[\end{aligned} \tag{1.1}$$