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ANALYSIS ON LOCAL DIRICHLET SPACES - II. UPPER GAUSSIAN ESTIMATES FOR THE FUNDAMENTAL SOLUTIONS OF PARABOLIC EQUATIONS

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0. Introduction

For every $t \in \boldsymbol{R}$, let $\mathscr{E}_t$ be a regular, local Dirichlet form with common domain $\mathscr{F} \subset L^2(X,m)$ on a locally compact, separable Hausdorff space X. We study the behaviour of (global as well as local) solutions of the parabolic equation

$$L_t u = \tfrac{\partial}{\partial t} u \quad \text{on } \boldsymbol{R} \times X. \tag{0.1}$$

The (not necessarily selfadjoint) operators L_t on $L^2(X,m)$ are supposed to be associated with the (not necessarily symmetric) Dirichlet forms $\mathscr{E}_t$ on $L^2(X,m)$ according to

$$-(L_t u, v) = \mathscr{E}_t(u,v).$$

To simplify things, let us first of all consider the case where all the $\mathscr{E}_t$'s are symmetric and strongly local. In this case, the only assumption in the whole paper which is imposed on the forms $\mathscr{E}_t$ is the uniform parabolicity condition

$$k \cdot \mathscr{E}(u,u) \leq \mathscr{E}_t(u,u) \leq K \cdot \mathscr{E}(u,u). \tag{0.2}$$

Here $\mathscr{E}$ is a fixed symmetric and strongly local, regular Dirichlet form. In terms of $\mathscr{E}$ we define the intrinsic distance ρ on X which is *assumed* to reproduce the original topology on X. The main result of the first part of this paper is the following integrated upper Gaussian estimate.

Theorem 0.1. *The transition operators T_t^s, $s<t$, associated with* (0.1) *can be estimated as follows*

$$(T_t^s 1_A, 1_B) \leq \sqrt{m(A)} \cdot \sqrt{m(B)} \cdot \exp\left(-\frac{\rho^2(A,B)}{4K(t-s)}\right) \cdot \exp(-k\lambda(t-s)) \tag{0.3}$$