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MEHLER FORMULA AND CAPACITIES FOR INIFNITE DIMENSIONAL ORNSTEIN-UHLENBECK PROCESSES WITH GENERAL LINEAR DRIFT

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0. Introduction

Let E be a locally convex topological vector space and μ a mean zero Gaussian Radon measure on its Borel σ-algebra $\mathscr{B}(E)$. Define the corresponding *Mehler semigroup* by

$$p_tf(z):=\int f(e^{-t}z+\sqrt{1-e^{-2t}}z')\mu(dz'),\quad z\in E,\quad t>0, \tag{0.1}$$

where $f\colon E\to[0,\infty[$ is $\mathscr{B}(E)$-measurable. Let $C_{r,p},r>0,p>1$ denote the corresponding (r,p)-capacities defined via the Gamma transform of $(p_t)_{t>0}$ (cf. the beginning of Section 1 below). It is well-known that each $C_{r,p}$ is *tight*, i.e., there exist compact sets $K_n\subset E$, $n\in N$, such that $C_{r,p}(E\backslash K)\to 0$ as $n\to\infty$. This was first proved by H. Sugita [50] if E is a separable Banach space, and subsequently extended by *D.* Feyel and A. de La Pradelle [19] to more general cases. The first main result of this paper (Theorem 2.2) is that each $C_{r,p}$ is in fact *strongly tight*, i.e., the compact sets K_n, $n\in N$, can be chosen metrizable for every E as above. This result was first announced in [13]. In this paper we give a detailed proof (which is also different from and shorter than the one indicated in [13]). Our proof depends on the well-known result by H.Sato [45] and B.S.Tsirelson [51] that there exist metrizable compact sets $K_n\subset E$, $n\in N$, such that $\mu(E\backslash K_n)\to 0$ as $n\to\infty$.

Before we describe our second main result we note that if H denotes the reproducing kernel Hilbert space of μ then H is separable and $H\subset E$ continuously (and also densely if μ is non-degenerate). Reversing view-points we now start with a separable Hilbert space H and assume that we are given a nonnegative definite self-adjoint linear operator A with domain $D(A)$ on H. We construct another separable real Hilbert space E carrying a mean zero Gaussian probability measure μ whose covariance is specified by a given bounded linear operator B on H which commutes with $e^{-tA},t>0$, such that the following holds: $(e^{-tA})_{t>0}$ extends to a strongly continuous contraction semigroup on E (cf. Theorem 3.1; in fact the embedding $H\subset E$ is Hilbert-Schmidt). H will be dense in E so that this