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AN EXTENSION OF WHITNEY'S CONGRUENCE

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1. Introduction and Main results

Throughout this paper, we will work in the *PL* category, and all embeddings will be locally flat.

Let M be a connected and oriented 4-manifold, F a closed and connected surface of Euler characteristic $\chi(F)$. For a given embedding of F into $M(F \subset M)$, let $e(M,F)$ be the normal Euler number of it, and let $[F]$ be the element in $H_2(M;Z_2)$ represented by F in M. We are interested in the ralation between $e(M,F)$ and $[F]$. In the case of $M=S^4$, the following theorem is well-known.

Theorem 1.1 (H. Whitney [8]: Whitney's congruence). If $M=S^4$,

$$e(M,F)+2\chi(F)\equiv 0 \quad mod\ 4.$$

For some time, we assume that M is closed and $H_1(M;Z)=\{0\}$. We will define a Z_4-quadratic map q from $H_2(M;Z_2)$ to Z_4 as follows. By the assumption $H_1(M;Z)=\{0\}$, the mod 2-reduction map p_2 from $H_2(M;Z)$ to $H_2(M;Z_2)$ is surjective. For a given element α in $H_2(M;Z_2)$, we define $q(\alpha)$ by

$$q(\alpha)\equiv\tilde{\alpha}\circ\tilde{\alpha} \quad \text{mod } 4,$$

where $\tilde{\alpha}$ is an element of ${p_2}^{-1}(\alpha)$ and $\circ$ is the intersection form on $H_2(M;Z)$.

The well-definedness of q is easy to see, and q is Z_4-quadratic, i.e.,

$$q(\alpha+\beta)\equiv q(\alpha)+q(\beta)+2(\alpha\bullet\beta) \quad \text{mod } 4,$$

where $\bullet$ is (Z_2-valued) intersection form on $H_2(M;Z_2)$, and 2: $Z_2\to Z_4$ is the natural embedding.

Using the quadratic function q, we extend Theorem 1.1 as follows:

Theorem 1.2.

$$e(M,F)+2\chi(F)\equiv q([F]) \quad mod\ 4.$$