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ENDOMORPHISMS OF HOMOGENEOUS SPACES OF LIE GROUPS

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If H is a closed subgroup of a topological group G it is well-known that there is a bijection

$$\mathrm{Map}_G(G/H, G/H) \rightleftarrows (G/H)^H$$

which is actually a homeomorphism when the mapping space is equipped with compact-open topology. Homeomorphisms correspond to the subspaces

$$\mathrm{Homeo}_G(G/H) \rightleftarrows NH/H.$$

Our main purpose is to prove

Theorem. *If G is a Lie group and H is a closed subgroup then NH/H is open in $(G/H)^H$.*

In [tD, Ch. IV.1] Tammo tom Dieck defines a universal additive invariant $U(G)$ of pointed finite G-CW-complexes for arbitrary topological groups G and computes it for compact Lie groups. As a corollary we obtain that his result is valid for arbitrary Lie groups, too.

Corollary. *$U(G)$ is a free abelian group on elements $u(G/H^+)$ where H runs through a complete set of conjugacy classes of closed subgroups H in G for any Lie group G.*

The condition

$$(O) \quad NH/H \text{ is open in } (G/H)^H$$

was introduced in a study with Wolfgang Lück [LL] in order to define the equivariant Lefschetz class of a G-endomorphism $f\colon X \to X$ of a finite G-CW-complex.

The inverse images of the subspaces $NH/H \subset (G/H)^H \subset G/H$ are