Ballico, E. and Keen, C.
Osaka J. Math.
32 (1995), 155–163

ON SMOOTH K-GONAL CURVES WITH ANOTHER FIXED PENCIL

EDOARDO BALLICO and CHANGHO KEEM

(Received May 10, 1993)

Let M_g be the moduli scheme of complex smooth complete curves of genus g and $M_{g,k}$ its integral subvariety parametrizing the k-gonal curves. For any integer a with $k \leq a \leq g/2$, let $U(k,a;g)$ be the constructible subset of $M_{g,k}$ parametrizing the k-gonal curves with base point free, simple and complete g_a^1; in particular, note that the g_a^1 will not be composed with the g_k^1; hence we have $g \leq (a-1)(k-1)$. In section 2 we will consider $U(k,a;g)$ very briefly (using [1]).

Our main result is the following theorem.

Theorem 0.1. *Fix non negative integers g, k, a, u, r, with $4 \leq k \leq a \leq g/2$. Set $n := (a-1)(k-1) - g$ and assume $g \leq (a-1)(k-1)$ i.e. $n \geq 0$. Assume the following condition:*

$$3g \geq 2ak - 4a - 4k + 3 \tag{1}$$

Then there is an integral family T of genus g curves in $U(k,a;g)$ such that for a general $C \in T$ (with g_k^1 and g_a^1 as linear systems) we have

$$\begin{aligned}\dim |rg_k^1 + ug_a^1| = (r+1)(u+1) + n - (k-u-1)(a-r-1) + \\ + \max(0, \max(k-u-1, 0) \cdot \max(a-r-1, 0) - n) - 1\end{aligned} \tag{2}$$

In particular $|rg_k^1 + ug_a^1|$ is not special if $n \geq (k-1-u)(a-1-r)$.

As will be clear from the proof of 0.1 (or essentially just by Riemann-Roch and Serre duality) the value given by (2) corresponds to the minimum possible value (which by semicontinuity will then be the value for $|rg_k^1 + ug_a^1|$ in an open subset of $U(k,a;g)$); indeed we will just prove the existence of one such curve, C, and then use semicontinuity to claim the same result for a general element in any component, T, of $U(k,a;g)$ containing C.

In section 3 we will prove Theorem 0.1. For a related statement (which gives the existence of large families in $U(k,a;g)$ for which the value for $|rg_k^1 + ug_a^1|$ is given and different from the one in eq. (2)), see Theorem 3.1. The proof of 0.1 is just a very classical game with singular curves contained as divisors of type