ARTINIAN RINGS RELATED TO RELATIVE ALMOST PROJECTIVITY

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Let R be an artinian ring. We consider the following condition: if eR/A is fR/B-projective (resp. N-projective for an R-module N), then every submodule M' of eR/A is fR/B-projective (resp. N-projective), where e and f are primitive idempotents. We have shown in [7] that R satisfies the above condition for any eR/A and any fR/B if and only if R is a hereditary ring with $J^2 = 0$. In this paper we consider a weaker condition: if eR/A is N-projective, then M' is almost N-projective where i): N is local and ii): N is a direct sum of local modules, respectively. In the second section we shall study QF, QF-2, and QF-3 rings with the above weaker condition, respectively. We study right almost hereditary rings with $J^2 = 0$ in the third section.

In a forthcoming paper we shall give a charaterization of rings over which the weaker condition is satisfied when M and N are any R-modules.

1. Characterizations

We always assume that R is an associative artinian ring with identity and every module is a finitely generated and unitary right R-module. Moreover since we are interested in the structure of R, we may assume that R is basic.

Let M and N be any finitely generated R-modules. We have studied rings with the following properties (1) (4) in [3] and [7]:

(1) If M is N-projective, then M' is again N-projective for any submodule M' of M.

(2) If eR/B is fR/A-projective, then C/B is again fR/A-projective for any $C \supset B$, where e and f are primitive idempotents and $C \supset B$ (resp. A) are R-submodules of eR (resp. fR).

(3) $e = f$ in (2).

(4) If M is almost N-projective, then M' is again almost N-projective for any submodules M' of M.

Here we shall consider a weaker condition than (4).