

MARKOV PROCESSES ASSOCIATED WITH SEMI-DIRICHLET FORMS

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1. Introduction

Recently, in [1], [11] an analytic characterization of all (non-symmetric) Dirichlet forms (on general state spaces) which are associated with pairs of special standard processes has been proved extending fundamental results in [8], [9], [18], [5], [10] (cf. also the literature in [11]). These Dirichlet forms are called *quasi-regular* (cf. Section 3 below). The processes forming the pairs are in duality w.r.t. the reference (*speed*) measure of the Dirichlet form. From a probabilistic point of view, however, this duality is quite restrictive. It arises from the fact that a Dirichlet form by the definition in [1], [11] exhibits a contraction property in *both* of its arguments. More precisely, we recall that a coercive closed form $(\mathcal{E}, D(\mathcal{E}))$ on $L^2(E; m)$ (cf. Section 2 below) is called a *Dirichlet form* if for all $u \in D(\mathcal{E})$ we have $u^+ \wedge 1 \in D(\mathcal{E})$ and

$$(1.1) \quad \mathcal{E}(u + u^+ \wedge 1, u - u^+ \wedge 1) \geq 0$$

$$(1.2) \quad \mathcal{E}(u - u^+ \wedge 1, u + u^+ \wedge 1) \geq 0.$$

The purpose of this paper is to show that quasi-regularity is also sufficient and necessary for the existence of an associated special standard process if the given coercive closed form is merely a *semi-Dirichlet form*, i.e., only (1.1) (or (1.2)) holds. The existence of a (Hunt) process associated with a semi-Dirichlet form $(\mathcal{E}, D(\mathcal{E}))$ was first proved in [5] in the case where E is a locally compact separable metric space under much more stronger assumptions on $(\mathcal{E}, D(\mathcal{E}))$.

Let us now briefly describe the contents of the single sections of this paper in more detail. In section 2 we first prove a few new results for the (one sided) analytic potential theory of semi-Dirichlet forms which are needed later. Here we only require that E is a measurable space in contrast to earlier work on this subject (cf. [5], [2], [3], where e.g. the measure representation of potentials was crucial which could only be obtained because E was assumed to be locally compact). In particular, we give a new proof for the characterization of α -excessive