Sekine, J.
Osaka J. Math.
32 (1995), 71–95

THE HILBERT RIEMANNIAN STRUCTURE OF EQUIVALENT GAUSSIAN MEASURES ASSOCIATED WITH THE FISHER INFORMATION

JUN SEKINE

(Received January 11, 1993)

1. Introduction.

Rao has firstly introduced the Riemannian structure associated with the Fisher information matrix over a finite dimensional parametrized statistical model. He proposed the Riemannian distance as a measure of dissimilality between two probability measures. (cf. [2], for example.) In [1], Amari introduced a pair of dual affine connections with respect to the metric and discussed of the differential geometry of the space of a finite dimensional parametrized statistical model. It provides a differential geometrical meaning to statistical inference.

In the present paper, we realize the above idea for a family of equivalent (i.e.,mutually absolute continuous) Gaussian measures on a Banach space. Our main result is as follows.

Let B be a real separable Banach space and P be a centered gaussian measure on B', the topological dual of B (cf. [6]). The covariance of P naturally determines the Hilbert space H. i.e., for arbitrary x_1 and $x_2 \in B$, let

$$C^P(x_1, x_2) = \int_{B'} \langle x_1, \xi \rangle \langle x_2, \xi \rangle P(d\xi)$$

be the covariance operator of P where $\langle \cdot, \cdot \rangle$ denotes the natural pairing between B and B'. The completion of $(B, C(\cdot, \cdot))$ is a separable Hilbert space. We denote it by $(H, (\cdot, \cdot))$. The space B is continuously embedded in H, so the following relation is satisfied

$$B \underset{\rightarrow}{\subset} H \cong H' \underset{\rightarrow}{\subset} B'.$$

Let us denote

$$\bar{\Theta}_1 = \{A \in L^s_{(2)}(H);\ (I + A) \text{ is positive definite}\} \tag{1.1}$$