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ON LOCALIZATIONS OF A CLASS OF STRONGLY HYPERBOLIC SYSTEMS

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1. Introduction

In a previous paper, henceforth quoted as [7], necessary conditions have been obtained for a first order system L to be strongly hyperbolic. In patricular, these conditions assert that, at an involutive characteristic, say z^0, the dimension of $\mathrm{Ker}L(z^0)$ must be equal to the order (or the multiplicity) of z^0 (Corollary 1.4 in [7]). Then the Taylor expansion of L along $\mathrm{Ker}L(z^0)$ starts with a linear term L_{z^0}, called the localization at z^0 (Section 3), which would be the first candidate for approximations of L on $\mathrm{Ker}L(z^0)$.

Unfortunately, in general, the localization is not diagonalizable even if the original system is strongly hyperbolic and the characteristic is involutive, in contrast with constant coefficient case (see Lemma 8 in [8] and Example 4.1 below). Our aim in this paper is then to make more detailed studies on localizations at an involutive characteristic of strongly hyperbolic systems.

Our first result is concerned with the localization L_{z^0} at an involutive characteristic z^0 of order r of a strongly hyperbolic system L. Hence L_{z^0} is a $r\times r$ system. Then we show that every $(r-1)$-th minor of L_{z^0} vanishes of order $s-2$ at every characteristic of order s of L_{z^0} (Theorem 4.1). This means that the localization must satisfy the same necessary condition which is verified by the original strongly hyperbolic system (see Theorem 1.1 in [7]).

The second result is stated as: Let z^0, z^1 be characteristics of the original system L and of the localization L_{z^0} of order r and s respectively. Then, assuming that the characteristic set of L is an involutive C^∞ manifold, every $(r-1)$-th minor of L_{z^0} vanishes of order $s-1$ at z^1 if (z^0,z^1) is "involutive" (Theorem 5.1). In particular, the dimension of $\mathrm{Ker}L_{z^0}(z^1)$ is equal to s and hence $L_{z^0}(z^1)$ is diagonalizable. If the characteristic z^0 is non degenerate (Definition 3.3) and (z^0,z^1) is "involutive" for every characteristic z^1 of L_{z^0}, then the localization L_{z^0} is strongly hyperbolic, more precisely the coefficient matrices of L_{z^0} are simultaneously symmetrizable (Theorem 5.2). We also show that the same results hold under less restrictive assumptions on the characteristics, which though, are not coordinate free (Propositions 6.1 and 6.2). These results, applied to the constant coefficients case, generalize Theorem 1 in [8].