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ON A THETA PRODUCT FORMULA FOR THE SYMMETRIC A-TYPE CONNECTION FUNCTION

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1. Introduction

In this note we are concerned about a formula which gives a product expression for a sum of theta rational functions. This sum has already appeared in the connection formulae among symmetric A-type Jackson integrals (See [1], [2]).

Let $q\in C$, $|q|<1$ be the elliptic modulus. We shall use frequently the Jacobi elliptic theta function $\theta(u)=(u)_\infty(q/u)_\infty(q)_\infty$, where $(u)_\infty=\Pi_{\nu=0}^{\infty}(1-q^\nu u)$. Let $\alpha_1,\alpha_2,\cdots,\alpha_n$, β, γ and γ' be complex numbers such that $\alpha_j=\alpha_1+(j-1)(\gamma'-\gamma)$ and $\gamma+\gamma'=1$. The symmetric group of n-th degree $\mathscr{G}_n$ acts on a function $f(t)$ on the n dimensional algebraic torus $(C^*)^n$ as $\sigma f(t)=f(\sigma^{-1}(t))=f(t_{\sigma(1)},\cdots,t_{\sigma(n)})$ for $t=(t_1,\cdots,t_n)\in(C^*)^n$.

Let $\{U_\sigma(t)\}_{\sigma\in\mathscr{G}_n}$ be the theta rational functions on $(C^*)^n$ defined as follows,

$$U_\sigma(t)=\Pi_{\substack{1\le i<j\le n\\ \sigma^{-1}(i)>\sigma^{-1}(j)}}\left(\frac{t_j}{t_i}\right)^{\gamma-\gamma'}\frac{\theta(q^\gamma t_j/t_i)}{\theta(q^{\gamma'}t_j/t_i)}. \tag{1.1}$$

These are pseudo-constants and define one-cocycle of $\mathscr{G}_n$ with values in C^*,

$$U_{\sigma\sigma'}(t)=U_{\sigma'}(t)\cdot\sigma U_{\sigma'}(t)\qquad\text{and}\qquad U_e(t)=1 \tag{1.2}$$

for all $\sigma,\sigma'\in\mathscr{G}_n$ (e denotes the identity).

Let $\varphi(x), x=(x_1,\cdots,x_n)\in(C^*)^n$, be the theta rational function

$$\varphi(x)=\Pi_{j=1}^{n}x_j^{\alpha_j}\frac{\theta(q^{\alpha_j+\cdots+\alpha_n+\gamma+1}x_j/x_{j-1})}{\theta(q^{\gamma+1}x_j/x_{j-1})} \tag{1.3}$$

for $x_0=q^\gamma$. Consider the generalized alternating sum with the weight $\{U_\sigma^{-1}(x)\}_{\sigma\in\mathscr{G}_n}$ as follows,

$$\tilde{\varphi}(x)=\sum_{\sigma\in\mathscr{G}_n}\sigma\varphi(x)\cdot\operatorname{sgn}(\sigma)\cdot U_\sigma(x)^{-1}. \tag{1.4}$$

It has the equivariant property