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THE CHERN CHARACTER OF THE SYMMETRIC SPACE $SU(2n)/SO(2n)$

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0. Introduction

For $n \geq 1$, let t: $SU(n) \to SU(n)$ be the map defined by $t(x) = \bar{x}$ for $x \in SU(n)$, where $\bar{x}$ is the complex conjugate of a unitary matrix x. The natural inclusion $\boldsymbol{R} \subset \mathbf{C}$ yields a monomorphism i_1: $SO(n) \to SU(n)$ of topological groups. Clearly $i_1(SO(n)) = \{x \in SU(n) \mid t(x) = x\}$. So the quotient space $SU(n)/i_1(SO(n))$, which we abbreviate to $SU(n)/SO(n)$, forms a compact symmetric space. It is denoted by AI (of rank $n-1$) in É. Cartan's notation. In this paper we compute its Chern character

$$ch\colon K^*(SU(2n)/SO(2n)) \to H^{**}(SU(2n)/SO(2n);\ \boldsymbol{Q}),$$

while that of $SU(2n+1)/SO(2n+1)$ has been described in [8].

1. K-rings

In this section we collect some results on K-theory of related spaces needed in the sequel.

Let G be a compact Lie group. Then the complex representation ring $R(G)$ forms a λ-ring. For each integer $k \geq 0$, let λ^k: $R(G) \to R(G)$ be the k-th exterior power operation. The following is a known result: see [4, Chapter 13] or [9, Chapter 4].

Proposition 1. *For $n \geq 2$, put $\lambda_1 = [\boldsymbol{C}^n] \in R(SU(n))$ and let $\lambda_k = \lambda^k(\lambda_1)$. Then*

$$R(SU(n)) = \boldsymbol{Z}[\lambda_1, \lambda_2, \cdots, \lambda_{n-1}],$$

where $\lambda_0 = 1$ and $\lambda_n = 1$.

Proposition 2. *t^*: $R(SU(n)) \to R(SU(n))$ satisfies*

$$t^*(\lambda_k) = \lambda_{n-k} \quad \textit{for} \quad k = 1, \cdots, n-1.$$