Kawakami, T.
Osaka J. Math.
31 (1994), 831–835

NONISOMORPHIC ALGEBRAIC MODELS OF NASH MANIFOLDS AND COMPACTIFIABLE C^∞ MANIFOLDS

TOMOHIRO KAWAKAMI

(Received March 5, 1993)

1. Introduction

A well-known theorem of A. Tognoli [9] asserts that every compact C^∞ submanifold M of $\boldsymbol{R}^n$ with 2 dim $M+1\leq n$, one can find a C^∞ imbedding $e:M\to\boldsymbol{R}^n$, arbitrarily close in the C^∞ topology to the inclusion map $M\to\boldsymbol{R}$, such that $e(M)$ is a nonsingular algebraic subset of $\boldsymbol{R}^n$. In particular, M admits an algebraic model. Here an algebraic model of M means a nonsingular algebraic subset of some Euclidean space diffeomorphic to M.J. Bochnak and W. Kucharz showed in [4] that M has a continuous family of birationally nonequivalent algebraic models when M is a connected closed manifold with dim $M\geq 1$. In this paper, we consider algebraic models of a given affine Nash manifold and a given compactifiable C^∞ manifold. Here we say that a C^∞ manifold M is *compactifiable* if there exists a compact C^∞ manifold Y with boundary such that M is C^∞ diffeomorphic to the interior of Y. M. Shiota proved in [8, Remark 6.2.11] that any affine Nash manifold admits an algebraic model. We prove that either any Nash manifold or any compactifiable C^∞ manifold have an infinite family of birationally nonequivalent algebraic models. More precisely, we prove the following.

Theorem 1. *Each affine Nash manifold M with dim $M\geq 1$ has an infinite family of nonsingular algebraic subsets $\{X_n\}_{n\in N}$ of some Euclibean space such that each X_n is Nash diffeomorphic to M and that X_n is not birationally equivalent to X_m for $n\neq m$.*

Theorem 2. *Every compactifiable C^∞ manifold M with dim $M\geq 1$ has an infinite family of nonsingular algebraic subsets $\{X_n\}_{n\in N}$ of some Euclidean space such that each X_n is C^∞ diffeomorphic to M and that X_n is not birationally equivalent to X_m for $n\neq m$.*

Theorem 2 is a refinement of [3, Corollary 3.3]. We have next