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DADE'S CONJECTURE FOR TAME BLOCKS

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0. Introduction

Let G be a finite group, B a p-block of G, where p is a prime. In [5], Dade conjectured that the alternating sum of the numbers of irreducible characters of certain heights in some blocks of subgroups of G related to B vanishes. (See Section 1 below.) Moreover, he showed that the conjecture holds for blocks with cyclic defect groups and for any blocks of the first Janko group and the smallest Mathieu group. (See Sections 9, 10 and 11 of [5].) The cyclic defect group case can be handled since the structure of such blocks is well known by Dade's work. So, the answer to the conjecture in this case is completely due to him. On the other hand, by virtue of [4], [8] and [6], the structure of tame blocks, that is, 2-blocks whose defect groups are dihedral, quaternion or quasidihedral, is also well known. Thus, one could expect that the conjecture can also be proved in these cases. In fact, the purpose of the present paper is to show that one form of Dade's conjectures, whose concern is extended to the number of ordinary irreducible characters invariant under the action of given automorphisms, holds for tame blocks. (See Section 1.) Thus, for example, the principal 2-block of the smallest Mathieu group, which is treated concretely in Section 11 of [5], is just an example of our case.

Notations and terminologies are standard. See for example [7] and [2]. For any fixed p-block B of a finite group G, and any subgroup H of G, we denote by $\mathrm{Bl}(H, B)$ the set of those p-blocks b of H which satisfy $b^G = B$. The sets of ordinary irreducible characters and Brauer irreducible characters in B are denoted by $\mathrm{Irr}(B)$ and $\mathrm{IBr}(B)$, respectively. The cardinalities of $\mathrm{Irr}(B)$ and $\mathrm{IBr}(B)$ are denoted by $k(B)$ and $l(B)$ as usual. For $\chi \in \mathrm{Irr}(B)$, we denote by $d(\chi)$ the biggest integer m such that p^m divides $|G|/\chi(1)$. Thus the sum of $d(\chi)$ and the height of χ gives the defect $d(B)$ of B. In this paper, a p-chain means a chain C: $P_0 < P_1 < \cdots < P_n$ of p-subgroups of G with $P_0 = O_p(G)$. The above n is called the lenth of C. If all P_i's are elementary abelian, then it is called an elementary chain.

This paper is organized as follows. After stating Dade's conjecture