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ON G-h-COBORDISMS BETWEEN G-HOMOTOPY SPHERES

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1. Introduction

We work in the smooth category, and G will be a finite group in the present paper. This paper is concerned with free G-actions on homotopy spheres.

Let us recall Milnor's theorem:

Theorem 1.1 ([6; Corollary 12.13]). *Any h-cobordism W between lens spaces L and L' must be diffeomorphic to $L\times[0,1]$ if the dimension of L is greater than or equal to* 5.

Let V be a unitary G-representation space of complex dimension n. When we consider a unit sphere $S(V)$ in V, we call it *a linear G-sphere* or we say that it has *a linear G-action.* In particular, if the G-action is free, $S(V)$ is called *a free linear G-sphere.* We see that Theorem 1.1 is put in another form as follows:

Theorem 1.2. *Let $S(V)$ and $S(V')$ be free linear G-spheres of dimension $2n-1\geqq 5$. If G is cyclic and W is a G-h-cobordism between $S(V)$ and $S(V')$, then W must be G-diffeomorphic to $S(V)\times I$, where $I=[0,1]$.*

As a generalization of Theorem 1.2, we proved the following result and gave some examples in [11].

Proposition 1.3 ([11; Proposition 3.1]). *Let G be a finite group such that $SK_1(\boldsymbol{Z}[G])=0$. Then the following hold*:

(1) *If X is a free G-homotopy sphere of dimension $2n-1\geqq 5$, any G-h-cobordism W between X and itself must be G-diffeomorphic to $X\times I$.*
(2) *If $S(V)$* and *$S(V')$ are free linear G-spheres of dimension $2n-1\geqq 5$, any G-h-cobordism W' between $S(V)$ and $S(V')$ must be G-diffeomorphic to $S(V)\times I$.*