Yoshikawa, K.
Osaka J. Math.
31 (1994), 497–522

AN ENUMERATION OF SURFACES IN FOUR-SPACE

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(Received February 5, 1993).

1. Introduction

In this paper, we consider the enumeration problem of knotted surfaces in Euclidean 4-space R^4. In classical case, there are many works on that of classical knots and links in Euclidean 3-space R^3 since the 19th century (e.g. [12], [13], [14], [19]). Particularly, J.H. Conway gave a notation of classical knots and links in R^3, the so called tangle, which is suitable for machine computation, and he listed all classical knots of at most 11 crossings and all classical links of at most 10 crossings [4]. In 4-dimensional case, the author made a table of knotted surfaces in R^4 with ch-index 10 or less [22], which will appear as an appendix of this paper. (The ch-index of a knotted surface will be defined in Section 2.) The purpose of this paper is to show a method of enumerating knotted surfaces in R^4 which is used to make the table in [22].

We work in the piecewise-linear (or smooth) category. By a surface in R^4 (or S^4), we mean a closed and locally-flat (possibly disconnected or non-orientable) surface embedded in R^4 (or S^4) unless otherwise stated. In Section 2, we study a diagram of a surface F in R^4 and define the ch-index of F, which is a numerical invariant of the knot type of surfaces in R^4 and has a property that, for each $n \geq 0$, the number of the knot types of non-splittable surfaces in R^4 with ch-index n is finite. Thus it is suitable for enumeration of surfaces in R^4. In Section 3, we introduce a graph of a surface in R^4. In Section 4, we explain how to list all surfaces with ch-index n in R^4 for each n by using graphs. In Section 5, we give some remarks on surfaces of [22]. Appendix contains two tables. The first is the table of surfaces in R^4 which was given in [22]. The other is that of their groups and first elementary ideals.

2. Diagrams of surfaces

In order to list surfaces in 4-space, we first need a convenient way of describing such surfaces. We use Fox's motion picture representation