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UNORIENTED ANALOGUE OF ELLIPTIC GENERA

Dedicated to Professor Haruo Suzuki on his sixtieth birthday

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0. Introduction

Serge Ochanine [8] determined the ideal $\mathscr{I}$ of $\Omega^{SO}\otimes\mathbf{Q}$, generated by all the oriented bordism classes $[CP(\xi^{2n})]$ of the total spaces of odd-dimensional complex projective space bundles over closed oriented smooth manifolds. He showed, in particular, that a multiplicative genus

$$\phi\ :\ \Omega^{SO}\otimes\mathbf{Q}\ \rightarrow\ \mathbf{Q}$$

annihilates the ideal $\mathscr{I}$ if and only if its logarithm $g(u)$ is formally given by an elliptic integral

$$g(u)=\int_0^u \frac{du}{\sqrt{1-2\phi([CP_2])u^2+\phi([H_{3,2}])u^4}}\ .$$

In this note we examine an unoriented bordism analogue of the above results. Namely, let $\mathscr{I}^R$ be the ideal of the unoriented bordism ring $\mathfrak{N}_*$, consisting of all the bordism classes $[RP(\zeta^{2m})]$ of the total spaces of odd-dimensional real projective space bundles over closed manifolds. We will prove that a multiplicative genus

$$\phi\ :\ \mathfrak{N}_*\rightarrow\ \mathbf{Z}_2=\mathbf{Z}/(2)$$

annihilates $\mathscr{I}^R$ if and only if its logarithm $g(u)$ is given by

$$\begin{aligned} g(u)&=\int_0^u \frac{du}{\sqrt{1-2\phi([RP_2])u^2+\phi([H^R_{3,2}])u^4}} \\ &=\int_0^u \frac{du}{1+\phi([RP_2])u^2}\ , \end{aligned}$$

which means $\phi([RP_{2i}])=\phi([RP_2])^i$ for $i\geqq 2$.