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EQUIVARIANT CRITICAL POINT THEORY AND IDEAL-VALUED COHOMOLOGICAL INDEX

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Introduction

We develop an equivariant critical point theory for differentiable G-functions on a Banach G-manifold with the aid of ideal-valued cohomological index theory, where G is a compact Lie group. We obtain a lower bound for the number of critical orbits with values in a given interval $(a,b]=\{t\in\boldsymbol{R}|a<t\leq b\}$ and for the number of critical values in $(a,b]$. We also obtain cohomological information about the topology of the critical set K of a G-function, which says a lot more about K than that obtained by using the Lusternik-Schnirelmann category.

The Lusternik-Schnirelmann category is a numerical homotopical invariant which gives a lower bound for the number of critical points (see for example [16], [17]), and this category is successfully extended to the equivariant setting [2], [3], [5], [6], [7], [15]. Ideal-valued cohomological index theory also gives important information about the existence of critical points [8], [9], [10]. The index theory in these papers is a priori in the equivariant setting and contains the nonequivariant (absolute) setting as trivial case.

In their paper [6] M. Clapp and D. Puppe developed an equivariant critical point theory using an equivariant Lusternik-Schnirelmann category. In the present paper we will develop one using an ideal-valued cohomological index theory which contains the nonequivariant setting as nontrivial case. We will obtain a type of results corresponding to their Theorem 1.1 of [6] and further results which are derived only from our theory.

Throughout this paper G always denotes a compact Lie group, and spaces considered are all paracompact Hausdorff. Let M be a Banach G-manifold of class at least C^1, i.e., M is a C^1 Banach manifold and G acts differentiably by diffeomorphisms. Let f: $M\to\boldsymbol{R}$ be a C^1 G-function, i.e., f is of class C^1 and satisfies $f(gx)=f(x)$ for all $x\in M$ and $g\in G$. Let $K=\{x\in M|df_x=0\}$ the critical set of f, $M_c=f^{-1}(-\infty,c]$ and $K_c=K\cap f^{-1}(c)$ for any $c\in\boldsymbol{R}$.

If $x\in M$ is a critical point of f, then every point of $Gx=\{gx|g\in G\}$