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A GENERALIZATION OF A THEOREM OF MILNOR

Dedicated to Professor Seiya Sasao on his 60th birthday

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1. Introduction

We work in the smooth category with free actions by groups in the present paper. Let us recall Milnor's theorem:

Theorem 1.1 ([6; Corollary 12.13]). *Any h-cobordism W between lens spaces L and L' must be diffeomorphic to $L\times[0,1]$ if the dimension of L is greater than or equal to 5.*

Let $\boldsymbol{Z}_m$ be the cyclic group of order m. Then we see that Theorem 1.1 is put in another way as follows:

Theorem 1.2. *Let $S(V)$ and $S(V')$ be free linear $\boldsymbol{Z}_m$-spheres of dimension $2n-1\geqq 5$. Then any $\boldsymbol{Z}_m$-h-cobordism W between $S(V)$ and $S(V')$ must be $\boldsymbol{Z}_m$-diffeomorphic to $S(V)\times I$, where $I=[0,1]$.*

Let R be a ring with unit, G a finite group. Put $GL(R)=\varinjlim GL_n(R)$ and $E(R)=[GL(R),GL(R)]$ the commutator subgroup of $GL(R)$. Then $K_1(R)$ denotes the quotient group $GL(R)/E(R)$. Let $\boldsymbol{Z}$ be the ring of integers and $\boldsymbol{Q}$ the ring of rational munbers. Let $\boldsymbol{Z}[G]$ and $\boldsymbol{Q}[G]$ denote the group rings of G over $\boldsymbol{Z}$ and $\boldsymbol{Q}$. The Whitehead group of G is the quotient group

$$Wh(G)=K_1(\boldsymbol{Z}[G])/<\pm g{:}g\in G>.$$

The natural inclusion map $i{:}GL(\boldsymbol{Z}[G])\rightarrow GL(\boldsymbol{Q}[G])$ gives rise to a group homomorphism $i_*{:}K_1(\boldsymbol{Z}[G])\rightarrow K_1(\boldsymbol{Q}[G])$. Then $SK_1(\boldsymbol{Z}[G])$ is defined by setting

$$SK_1(\boldsymbol{Z}[G])=\ker[i_*{:}K_1(\boldsymbol{Z}[G])\rightarrow K_1(\boldsymbol{Q}[G])].$$

In [15], C.T.C. Wall showed that $SK_1(\boldsymbol{Z}[G])$ is isomorphic to the torsion