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GENERALIZED ROCHLIN INVARIANTS OF FIXED POINT SETS

MIKIYA MASUDA AND REINHARD SCHULTZ

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Introduction

An action of a group G on a space X is said to be *semifree* if for each $x \in X$ either x is fixed under every element of G or else x is not fixed by any element of G except the identity. During the nineteen sixties and seventies it became apparent that the techniques of differential topology had numerous applications to differentiable actions of compact Lie groups (*cf.* [5], [7], [23]). In particular, these and previously developed techniques yielded considerable information on semifree differentiable actions of S^1 and S^3 on spheres. One result was a complete description of the homeomorphism types of the possible fixed point sets. Specifically, these are all closed manifolds with the same integral homology as a sphere of some appropriate dimension (see [12, Ch. V, §4]). On the other hand, questions about the diffeomorphism types of the fixed point sets are more difficult to answer. In this paper we shall prove a result (Theorem B below) that complements previous work on the smooth realization question; this is a special case of a more general result (Theorem A) relating the diffeomorphism type of the fixed point set to the diffeomorphism type of the ambient manifold. Although evidence suggests that an analog of Theorem B holds for semifree S^1-actions (see Proposition 3.2 and [27]), the proof of such an analog seems likely to require additional input. The proofs of Theorems A and B involve a higher dimensional analog of the well known *Rochlin invariant* for closed homology 3-spheres (*e.g.*, see [9], where it is called the μ-invariant).

1. Statement of main results

Let M be a closed oriented manifold such that $H_*(M;\mathbf{Z}_2) \approx H_*(S^n;\mathbf{Z}_2)$;

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