Sakai, K.
Osaka J. Math.
31 (1994), 373-386

PSEUDO-ORBIT TRACING PROPERTY AND STRONG TRANSVERSALITY OF DIFFEOMORPHISMS ON CLOSED MANIFOLDS

KAZUHIRO SAKAI

(Received February 8, 1993)

1. Introduction

We are interested in the dynamical property of a diffeomorphism f having the pseudo-orbit tracing property of a closed manifold M. Let d be a metric for M. A sequence of points $\{x_i\}_{i\in\boldsymbol{Z}}$ of M is called a *δ-pseudo-orbit* of f if $d(f(x_i), x_{i+1})<\delta$ for $i\in\boldsymbol{Z}$. A sequence $\{x_i\}_{i\in\boldsymbol{Z}}$ is said to be *f-ε-traced* by $y\in M$ if $d(f^i(y), x_i)<\varepsilon$ for $i\in\boldsymbol{Z}$.

We say that f has the *pseudo-orbit tracing property* (abbrev. **POTP**) if for every $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit of f can be f-ε-traced by some point.

In [5] Robinson proved that every Axiom A diffeomorphism satisfying strong transversality has **POTP**. Thus it will be natural to ask whether **POTP** implies Axiom A and strong transversality. For this problem we have partial results that are answered in [4] for dim $M=2$ and in [7] for dim $M=3$. However we have no answer for higher dimensions.

Our aim is to prove the following

Theorem. *The C^1 interior of all diffeomorphisms having* **POTP** *of a closed manifold M, $\mathcal{P}(M)$, coincides with the set of all Axiom A diffeomorphisms satisfying strong transversality.*

We say that f has the C^1 *uniform pseudo-orbit tracing property* (abbrev. C^1-**UPOTP**) if there is a C^1 neighborhood $\mathcal{U}(f)$ of f with the property that for $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit of $g\in\mathcal{U}(f)$ is g-ε-traced by some point. Since every Axiom A diffeomorphism satisfying strong transversality has C^1-**UPOTP** (see [6, Theorem]), if we establish our theorem, then the following corollary is obtained.

Corollary. *The set of all diffeomorphism having C^1-***UPOTP** *is characterized as the set of all Axiom A diffeomorphisms satisfying strong transversality.*