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HOMOGENEOUS COMPLETE INTERSECTION HODGE ALGEBRAS ON SIMPLICIAL COMPLEXES

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Introduction

Since De Concini-Eisenbud-Procesi [1] defined Hodge algebra, two special classes have been studied, one of which is an ordinal Hodge algebra and the other is a square-free Hodge algebra. An ordinal Hodge algebra (=algebra with straightening laws, ASL, for short) have been investigated in detail and we know that an ASL reflects strongly a nature of a poset.

On the other hand, let A be a square-free Hodge algebra. By [1], we can associate to A a unique simplicial complex Δ. Then A should accordingly reflect a nature of Δ. We call A a Hodge algebra on the simplicial complex Δ. The purpose of the present article is to classify the simplicial complex on which there exists a homogeneous complete intersection Hodge algebra of dimension ≤ 3. We often employ the arguments in [5].

In §1, we recall the definition of Hodge algebra and elementary definitions in topology. In §2, we give a classification of simplicial complexes Δ when there exists a homogeneous Hodge K-algebra on Δ which is a complete intersection. Its proof is given in §3.

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1. Preliminaries

Let Δ be a simplicial complex and let H be the set of vertices of Δ. We call an element of $\boldsymbol{N}^H$ a *monomial* on H, where $\boldsymbol{N}$ is the nonnegative integers and $\boldsymbol{N}^H$ is the set of $\boldsymbol{N}$-valued functions on H. Given two monomials L, M on H, we can define a product LM by assigning $LM(x)=L(x)+M(x)$ to $x\in H$. The *support* of a monomial M is the subset Supp $M=\{x\in H;\ M(x)\neq 0\}$. We define Σ_Δ by

$$\Sigma_\Delta=\{M\in \boldsymbol{N}^H;\ \text{Supp } M \text{ does not belong to } \Delta\},$$