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MINIMAL THICKNESS AND UNIQUENESS OF KERNEL FUNCTIONS FOR THE HEAT EQUATION IN SEVERAL VARIABLES

Dedicated to Professor Mitsuru Nakai on his 60th birthday

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1. Introduction

Let $\boldsymbol{R}^{n+1}=\boldsymbol{R}^n\times\boldsymbol{R}$ be the $(n+1)$-dimensional Euclidean spase $(n\geq 1)$. We consider the heat equation

$$Lu:=\frac{\partial u}{\partial t}-\Delta u=0$$

and its nonnegative solutions (called parabolic functions). For an unbounded domain Ω in $\boldsymbol{R}^{n+1}$, a nonnegative parabolic function u in Ω is called a kernel function at infinity (resp. at a point $(y,s)\in\partial_p\Omega$) if u is not identically equal to zero and if u vanishes continuously on $\partial_p\Omega$ (resp. on $\partial_p\Omega\backslash\{(y,s)\}$), where $\partial_p\Omega$ denotes the parabolic boundary of Ω

We study the existence and uniqueness of kernel functions for the domains of the following form:

$$\Omega_\alpha(D)=\{(x,t)\in\boldsymbol{R}^n\times\boldsymbol{R};\ t<0,\ (-t)^{-\alpha}x\in D\},$$

where $\alpha\in\boldsymbol{R}$ and D is a bounded starlike Lipschitz domain in $\boldsymbol{R}^n$ with center 0, that is, D is starlike with center 0 and for every point $x_0\in\partial D,D$ is defined by a Lipschitz graph in some neighborhood of x_0 such that the ray $x_0 0$ is its axis(see [3,p. 513]).

J.T. Kemper [5] has studied kernel functions at finite boundary points, but our concern is ones at infinity, as discussed in [7], [8] and [4]. It has been shown that $\Omega_\alpha(D)$ has a unique kernel function at infinity if $n=1$, $\alpha<1$ ([8])and if $n\geq 1$, $\alpha\leq 1/2$ ([7]). Here we use the convention

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