Nakazawa, N.
Osaka J. Math.
31 (1994), 291–329

ASYMPTOTIC EXPANSION OF THE BERGMAN KERNEL FOR STRICTLY PSEUDOCONVEX COMPLETE REINHARDT DOMAINS IN C^2

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(Received October 18, 1991)
(Revised July 13, 1993)

Introduction

This paper is concerned with the asymptotic expansion, due to Fefferman [2], of the Bergman kernel for a strictly pseudoconvex domain. Restricting ourselves to the class of complete Reinhardt domains in $\boldsymbol{C}^2$, we consider the symbol of a pseudodifferential operator which represents the singularity of the Bergman kernel. We give an integral representation of that symbol (see Theorem 2 in Section 1). By using that integral representation, we identify six coefficients of Fefferman's asymptotic expansion (see Theorems 1 and 1′ in Section 1).

Given a bounded strictly pseudoconvex domain Ω in $\boldsymbol{C}^N$ with C^∞ boundary $\partial\Omega$, we consider the Bergman kernel $K(z)$ for $z\in\Omega$, which is restricted to the diagonal of $\Omega\times\Omega$. Let $\lambda\in C^\infty(\bar{\Omega})$ be a negatively signed defining function of Ω in the sense that $\lambda<0$ in Ω and $|\text{grada}\,\lambda|>0$ on $\partial\Omega$. Let us recall a classical result of Hörmander [5] asserting that

$$(0.1)\qquad \lim_{z\to z_0}[-\lambda(z)]^{N+1}K(z)=\frac{N!}{\pi^N}J[-\lambda](z_0)>0 \qquad \text{for} \qquad z_0\in\partial\Omega,$$

where $J[-\lambda]$ denotes the Levi determinant defined by

$$J[-\lambda]=(-1)^{N+1}J[\lambda]=-\det\begin{pmatrix}\lambda & \partial\lambda/\partial\bar{z}_k\\ \partial\lambda/\partial z_j & \partial^2\lambda/\partial z_j\partial\bar{z}_k\end{pmatrix}.$$

Fefferman [2] refined this result by showing that

$$(0.2)\qquad K(z)=\frac{N!}{\pi^N}J[\lambda]\left(\frac{\varphi(z)}{\lambda(z)^{N+1}}+\psi(z)\log[-\lambda(z)]\right)$$

with $\varphi,\ \psi\in C^\infty$ near $\partial\Omega$. If one considers the Taylor expansions