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A DOMAIN MONOTONICITY PROPERTY OF THE NEUMANN HEAT KERNEL

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1. Introduction

Throughout this paper $p^{\Omega}(t, x, y)$ denotes the Neumann heat kernel of a bounded euclidean domain $\Omega \subset \boldsymbol{R}^d$ with the Neumann boundary condition. By definition, $p^{\Omega}(t, x, y)$ is the fundamental solution of the heat operator $L=\partial/\partial t-\Delta/2$ (Δ is the Laplace operator) with the Neumann boundary condition, i.e., for fixed $x \in \Omega$, it is a function in (t, y) satisfying the equation

$$\begin{cases} Lp^{\Omega}(t, x, y) = 0\,, & (t, y) \in \boldsymbol{R}^+ \times \Omega\,, \\ \dfrac{\partial}{\partial n} p^{\Omega}(t, x, y) = 0\,, & (t, y) \in \boldsymbol{R}^+ \times \partial\Omega\,, \\ p^{\Omega}(0, x, y) = \delta_x(y)\,, & y \in \Omega\,. \end{cases}$$

Physically $p^{\Omega}(t, x, y)$ represents the temperautre distribution in Ω at time t and point y if a heat source of total capacity one is present at point x at time 0 with the assumption that the boundary $\partial\Omega$ is impervious to heat conduction (adiabatic boundary). From this interpretation of the Neumann heat kernel it was conjectured (see Chavel [2] and Kendall [7]) that if Ω is a smooth convex domain and D is another smooth domain containing Ω, then for all $(t, x, y) \in (0, \infty) \times \Omega \times \Omega$

$$p^{\Omega}(t, x, y) \geq p^{D}(t, x, y)\,.$$

While the counterpart of this conjecture for the Dirichlet heat kernel (absorbing boundary condition) is obvious by the maximum principle (without assuming the convexity of the smaller domain Ω), the conjecture stated above was recently proved to be false without further hypotheses on the domains Ω and D, see Bass and Burdzy [1].

The convexity of the inner domian is necessary because of the following basic asymptotic relation:

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