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STABLE-LIKE PROCESSES: CONSTRUCTION OF THE TRANSITION DENSITY AND THE BEHAVIOR OF SAMPLE PATHS NEAR $t=0$

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Introduction

Let $X=(X_t, \boldsymbol{P}_x; x\in\boldsymbol{R}^d)$ be a d-dimensional pure jump type Markov process associated with the operator $-(-\Delta)^{\alpha(x)/2}(0<\alpha(x)<2)$. Following Bass [1], we call it the stable-like process with exponent $\alpha(x)$. Under a mild regularity condition for $\alpha(x)$, the process is first constructed by Bass [1] and next by Tsuchiya [12]: Bass has done it by showing the uniqueness of solutions to the martingale problem for the operator and Tsuchiya by showing the pathwise uniqueness of solutions to a stochastic differential equation associated with the operator.

In this paper, we will show the existence of a transition density and local Hölder conditions for sample paths of the process X with smooth exponent $\alpha(x)$. For this aim, we want to adapt the theory of pseudo-differential operators to the operator $-(-\Delta)^{-\alpha(x)/2}$, but its symbol $-|\xi|^{\alpha(x)}$ is not smooth. Hence we consider the operator L_Φ which is obtained from $-(-\Delta)^{\alpha(x)/2}$ by cutting off the support of its integral kernel (i.e. Lévy measure) with a positive smooth function Φ (see Section 1 for the precise definition of L_Φ). There exists a pure jump type Markov process X_Φ associated with L_Φ in the same sense as the above. Since L_Φ can be regarded as a pseudo-differential operator of variable order, we introduce a class of such operators and provide the fundamental theorem for algebra and asymptotic expansion formula of their symbols. Next we prove that L_Φ satisfies the (H)-condition (see [7] p.83 for the (H)-condition). These facts allow us to construct a fundamental solution, in the sense of pseudo-differential operators, to the initial-value problem for the equation $\partial_t-L_\Phi=0$. Furthermore, we show that this fundamental solution has a smooth kernel and this gives a transition density of X_Φ. Using a localization argument, we see that X also has a transition density. Finally, using certain estimates for the symbol of the fundamental solution and expanding the method of Khintchine [6] and Blumenthal and Getoor [3], we obtain the local Hölder conditions for sample paths of X; this result is a natural extension of that of