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ON SPECTRA OF RANDOM SCHRÖDINGER OPERATORS WITH MAGNETIC FIELDS

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1. Introduction

In this paper, we investigate properties of spectra of random Schrödinger operators with magnetic fields. In particular we study the asymptotics of the density of states. On a probability space (Ω, P), we consider a 1-form valued random field $b=\sum_{j=1}^{d} b_{\omega}^{j}(x)dx^{j}$ $(\omega\in\Omega, x\in\boldsymbol{R}^{d})$ and a real valued random field $V=V_{\omega}(x)$ on $\boldsymbol{R}^{d}$. We assume that the pair $(db.(x), V.(x))_{x\in\boldsymbol{R}^{d}}$ is stationary and ergodic on $\boldsymbol{R}^{d}$. We assume further conditions on b and V later. On the space $L^{2}(\boldsymbol{R}^{d})$ of complex square integrable functions on $\boldsymbol{R}^{d}$, we consider the operator formally written as follows:

$$L(b_{\omega}, V_{\omega}) = -\frac{1}{2}\sum_{j=1}^{d}(\frac{\partial}{\partial x^{j}}-ib_{\omega}^{j}(x))^{2}+V_{\omega}(x) \qquad (i=\sqrt{-1}).$$

Under the assumptions, as same as in the case of $b=0$ (cf. [1]), it is easily seen that the spectra of $L(b_{\omega}, V_{\omega})$ are independent of ω except for the elements of a P-measure null set. Our purpose is to show that several properties of random Schrödinger operators without magnetic fields are extended to our case. In particular we consider the asymptotics of the density of states at the infimum of its support. As same as in cases of Pastur [14] and Nakao [13] (cf. Chapter VI of [3]), the problem can be reduced to study the asymptotics of $t\to\infty$ of the Laplace transform of the density of states, i.e.,

$$\int_{-\infty}^{\infty} e^{-\lambda t}n(d\lambda) = \left(\frac{1}{2\pi t}\right)^{d/2} E^{W\times P}\left[\exp\left(-i\int_{0}^{t} db(*) - \int_{0}^{t} V(w(s))ds\right)\middle| w(t)=0\right], \tag{1.1}$$

where w is a d-dimensional Wiener process starting at 0, $\int_{0}^{t} db(*)$ is a stochastic integral of the 2-form db (for the exact form, see (3.2) below) and $E^{W\times P}$ is the expectation with respect to the product measure of P and the Wiener measure. One of difficulties of these problems comes from the fact that the right hand side of (1.1) is an oscillatory integral (cf. [7]).