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CONWAY POLYNOMIALS OF PERIODIC LINKS

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Introduction

Geometric properties of knots and links in a 3-sphere S^3 often have effect on their polynomial invariants. Periodicity of knots and links is one of them. Therefore to study periodic knots and links, it is significant to investigate their polynomial invariants. In this paper, we consider the following situation: Let $L=K_1\cup\cdots\cup K_\mu$, $\mu\geq 1$ be an oriented link and B be a trivial knot with $B\cap L=\emptyset$. We consider the p-fold cyclic cover q_p: $S^3\to S^3$ branched over B, where $p\geq 2$. We denote the preimage of L and K_i by $\tilde{L}$ and $\tilde{K}_i$, respectively and call them the covering links of L and K_i, respectively. Let $\tilde{K}_i=K_{i1}\cup\cdots\cup K_{i\nu_i}$ be a ν_i-component link. We give K_{ij} the orientation inherited from K_i. Then $\tilde{L}=\tilde{K}_1\cup\cdots\cup\tilde{K}_\mu=K_{11}\cup\cdots\cup K_{1\nu_1}\cup\cdots\cup K_{\mu 1}\cup\cdots\cup K_{\mu\nu_\mu}$ has the unique orientation. In 1971, Murasugi [7] showed a relationship between the Alexander polynomials of L and $\tilde{L}$ for the case $\mu\nu_\mu=1$. Later, Hillman [4], Sakuma [10] and Turaev [12] extended Murasugi's result to the general case. Our goal in this paper is to give a relation between the Conway polynomials of L and $\tilde{L}$ using their results (Theorem 2). To do this, we sharpen their formulas by expressing in terms of the Conway potential function [2] whose existence is shown by Hartley [3] (Theorem 1). Although the Alexander polynomial is usually defined with the difference by a unit of a polynomial ring, the Conway potential function is uniquely defined as an element of a polynomial ring.

We denote the Conway potential functions of L, $B\cup L$ and $\tilde{L}$ by $\Omega_L(t_1,\cdots,t_\mu)$, $\Omega_{B\cup L}(s,t_1,\cdots,t_\mu)$ and $\Omega_{\tilde{L}}(t_{11},\cdots,t_{1\nu_1},\cdots,t_{\mu 1},\cdots,t_{\mu\nu_\mu})$, respectively, where t_i, s and t_{ij} correspond to K_i, B and K_{ij}, respectively. Then concerning the Conway potential functions of L and $\tilde{L}$, the following formula holds:

Theorem 1. *Let $L=K_1\cup\cdots\cup K_\mu$, $\mu\geq 1$ be an oriented μ-component link and B be a trivial knot with $B\cap L=\emptyset$. Let $\tilde{L}$ be the p-fold covering link of L, where $p\geq 2$. Then*

$$\Omega_{\tilde{L}}(t_1,\cdots,t_1,\cdots,t_\mu,\cdots,t_\mu)$$
$$=\sqrt{-1}^{(p-1)(1-lk(B,L))}\,\Omega_L(t_1,\cdots,t_\mu)\prod_{j=1}^{p-1}\Omega_{B\cup L}(\xi^j,t_1,\cdots,t_\mu),$$