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ON THE ADDITIVITY OF h-GENUS OF KNOTS

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Introduction

We say that $(V_1, V_2; F)$ is a Heegaard splitting of the 3-sphere S^3, if both V_1 and V_2 are handlebodies, $S^3=V_1\cup V_2$ and $V_1\cap V_2=\partial V_1=\partial V_2=F$. Then F is called a Heegaard surface of S^3.

Let K be a knot in S^3. Then it is well known that there exists a Heeagard surface of S^3 which contains K. Thus we define $h(K)$ as the minimum genus among all Heegaard surfaces of S^3 containing K, and we call it the h-genus of K. We note here that any two Heegaard surfaces of S^3 with the same genus are mutually ambient isotopic ([11]).

By the definition, it follows that $h(K)=0$ if and only if K is a trivial knot and that $h(K)=1$ if and only if K is a torus knot. Hence if $h(K)=1$ then K is prime. In this paper we show:

Theorem. *Let K_1 and K_2 be non-trivial knots in S^3. If $h(K_1\# K_2)=2$, then $h(K_1)=h(K_2)=1$.*

On the other hand, we show the following two propositions.

Proposition 1. *Let K_1 and K_2 be non-trivial knots in S^3 with $(1, 1)$-decompositions. Suppose neither K_1 nor K_2 are torus knots. Then $h(K_1)=h(K_2)=2$ and $h(K_1\# K_2)=3$.*

Here, we say that a knot K admits a (g, b)-decomposition, if there is a genus g Heegaard splitting $(V_1, V_2; F)$ of S^3 such that $V_i\cap K$ is a b-string trivial arc system in V_i $(i=1, 2)$ (cf. [2] and [6]).

REMARK. Since every 2-bridge knot admits a $(1, 1)$-decomposition, there are infinitely many knots satisfying the hypthesis of Proposition 1.

Proposition 2. *Let n be an integer greater than 1 and K_n the knot illustrated in* Figure 1. *Then $h(K_n)=3$ and $h(K_n\# K)=3$ for any 2-bridge knot K.*