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HALF NEARFIELD PLANES

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1. Introduction

Let Π be an affine translation plane of order p^n. Let Π admit an affine homology group of order k with center P and axis $0Q$ where PQ is the line at infinity, P, Q are infinite points and 0 is the zero vector (or any affine point). We shall say that Π admits *symmetric homology groups* provided there is also an affine homology group of order k with center Q and axis $0P$.

A nearfield plane is an affine translation plane of order p^n that admits symmetric homology groups of order p^n-1. Actually, if a translation plane admits one affine homology group of order p^n-1 then it admits symmetric homology groups of order p^n-1. Of course, there are many examples of translation planes that admit an affine homology group of order k that do not admit symmetric homology groups. For example, the j-planes of order q^2 (see [9] or [12]) admit homology groups of orders $q+1$ and $q-1$ but do not always admit symmetric homology groups of either order.

Let Σ denote a translation plane of even order 2^r that admits an affine elation group of order $2^r/2$. Then Jha, Johnson, and Wilke [7] have shown that there is also an elation group of order 2^r and, in this case, the plane is a semifield plane. Is a similar result valid for translation planes of odd order k that admit an affine homology group of order $(k-1)/2$? If yes, then there would be an affine homology group of order $k-1$ which would imply that the plane is a nearfield plane.

The theorem of Thas [18], [19], Bader, Lunardon [2] classifying the flocks of hyperbolic quadrics in PG$(3, q)$ plays a major part in the study undertaken herein.

Theorem 1.1. (Thas, Bader, Lunardon) *Let F be a flock of a hyperbolic quadric in PG$(3, q)$. Then either F is linear, a Thas flock or an irregular flock with $q=11, 23$, or 59.*

It is well known that corresponding to a flock of a hyperbolic quadric in PG$(3, q)$ is a translation plane with spread in PG$(3, q)$ (see the Thas-Walker construction, e.g. in [8]). Moreover, in Johnson [8], the following connection was noted: