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LEFT CELLS IN THE AFFINE WEYL GROUP $W_a(\tilde{D}_4)$

Dedicated to Professor R.W. Carter on his sixtieth birthday

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The cells of affine Weyl groups have been studied for more than one decade. They have been described explicitly in cases of type $\tilde{A}_n (n \geq 1)$ [13], [9] and of rank ≤ 3 [1], [4], [10]. But there are only some partial results for an arbitrary irreducible affine Weyl group [2], [7], [8], [16], [17]. In [18], we constructed an algorithm to find a representative set of left cells of a certain crystallographic group W in a given two-sided cell. This provides us a practicable way to describe the cell of more groups. In the present paper, we shall apply it to the case when W is the affine Weyl group $W_a(\tilde{D}_4)$ (or denoted by W_a for brevity) of type $\tilde{D}_4$. We shall give an explicit description for all the left cells of W_a by finding a representative set of left cells of W_a. Before this paper, Du Jie gave an explicit description for all the two-sided cells of W_a, but he was unable to find the left cells of this group [5]. Chen Chengdong recently described all the left cells of W_a in terms of certain special reduced expressions of elements [3]. Comparing with their results, our description on the cells of W_a is neater and easier expressable in nature. Moreover, by doing the above work, we develop some technical skill in performing the mentioned algorithm In particular, we could avoid any computation of non-trivial Kazhdan-Lusztig polynomials throughout this work.

The content of the present paper is organized as below. Section 1 is the preliminaries. Some basic concepts and results concerning our algorithm are stated there. In section 2, we introduce the alcove forms of elements of W_a and also state some properties of elements of W_a in terms of alcove forms, which are quite useful in the subsequent sections Then in sections 3–5, we apply our algorithm to find a representative set Σ of left cells of W_a. Finally, in section 6, we describe all the left cells of W_a by making use of the set Σ.

1. Preliminaries

1.1 Let $W=(W, S)$ be a Coxeter group with S its Coxeter generator set. Let

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